26 implies that at every instant the ratio of the electric field to the magnetic field of an e.m wave is equal to the speed of light.

Poynting Vector

E-M waves carry energy with them, when these waves pass through space, they transfer energy to the object in their path. The rate of flow of energy in an e.m. wave is given by a vector S called Poynting vector (named after J.H Poynting). It is given by

$$\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0} \quad \text{———} \quad (27)$$

for a plane e.m wave, $\vec{E} \perp \vec{B}$

$$S = \frac{EB}{\mu_0} \quad \text{———} \quad (28)$$

Unit of S is $J/sec/m^2$

Also

$$C = \frac{E}{B}$$

$$S = \frac{E^2}{\mu_0 C} = \frac{cB^2}{\mu_0} \quad \text{———} \quad (29)$$

We have the eqn for electric and magnetic fields as

$$\vec{E} = \vec{E}_0 \cos(kx - wt)$$

and

$$\vec{B} = \vec{B}_0 \cos(kx - wt)$$

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$$S = \left[\frac{E_0 B_0}{\mu_0}\right] \cos^2(kx - \omega t) \text{ — — } 30$$

If S_{av} is the average value of S taken over a complete cycle.

$$S_{av} = \frac{1}{T}\int_0^T S \cdot dt = \frac{1}{T}\int_0^T \frac{E_0 B_0}{\mu_0} \cos^2(kx - \omega t)\, dt$$

$$= \frac{1}{T} \times \frac{E_0 B_0}{\mu_0} \times \frac{T}{2} \qquad \left[\int_0^T \cos^2(kx-\omega t)\, dt = \frac{T}{2}\right]$$

$$S_{av} = \frac{E_0 B_0}{2\mu_0} = \frac{E_0^2}{2\mu_0 c} = \frac{c B_0^2}{2\mu_0} \text{ — } (31) \quad \frac{E_0}{B_0} = c$$

where

S_{av} is called the wave intensity. The instantaneous energy density associated with an electric field is given by

$$U_e = \frac{1}{2}\varepsilon_0 E^2 \text{ — — } (32)$$

Bcos energy is stored in magnetic field, then

$$U_B = \frac{1}{2} L I^2 = \frac{1}{2}\left[\frac{\mu_0 N^2 A}{l}\right]\left[\frac{Bl}{\mu_0 N}\right]^2 = \frac{1}{2}\frac{B^2}{\mu_0} Al \text{ — } (33)$$

∴ Energy stored / unit volume $= \frac{1}{2}\frac{B^2}{\mu_0}$

$$= \frac{1}{2}\frac{E^2}{c^2} \cdot \frac{1}{\mu_0} \qquad \left(\text{from } \frac{E}{B} = c\right)$$

$$= \frac{1}{2}\varepsilon_0 E^2 \qquad \left(\text{from } c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}\right) \quad (34)$$

The instantaneous energy density associated with a magnetic field is given by

$$U_m = \frac{B^2}{2\mu_0}$$

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$$\therefore U_m = \frac{(E/c)^2}{2\mu_0} = \frac{E^2/c^2}{2\mu_0} = \frac{E^2}{2\mu_0} \times (\mu_0 \varepsilon_0)$$

$$= \frac{1}{2}\varepsilon_0 E^2 = U_e \text{ ——— } (35)$$

(35) shows that for an e.m wave the instantaneous energy density associated with the magnetic field B equal to the instantaneous energy density associated with the electric field.

In other words in a given volume, the energy is shared equally by two fields.

If U is the total energy density, then

$$U = U_e + U_m = \varepsilon_0 E^2 = \frac{B^2}{\mu_0} \text{ ——— } 36.$$

If U_{av} is the average value of U, taken over a complete cycle, then,

$$U_{av} = \frac{1}{T}\int_0^T U \cdot dt = \frac{1}{T}\int_0^T \varepsilon_0 E^2 dt$$

$$= \frac{\varepsilon_0}{T}\int_0^T E_0^2 \cos^2(kx - \omega t)\,dt$$

$$= \frac{\varepsilon_0}{T} \cdot E_0^2 \cdot \frac{1}{2} \qquad \text{from } \left[\int_0^T \cos^2(kx-\omega t)\,dt = \frac{T}{2}\right]$$

$$U_{av} = \frac{1}{2}\varepsilon_0 E^2 \text{ ——— } (37)$$

Also $\dfrac{E_0}{B_0} = c$

$$\therefore U_{av} = \frac{B_0^2}{2\mu_0} \text{ ——— } (38)$$

$$U_{av} = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\frac{B_0^2}{\mu_0} \text{ ——— } (39)$$

from (31)

$$S_{av} = c \cdot \frac{1}{2} \frac{B_0^2}{\mu_0}$$

$$S_{av} = c \cdot U_{av} \quad \text{———} (40)$$

(40) Show that the intensity of an em wave is equal to the product of average energy density with the speed of light.

(Read on divergence & curl of a vector point)

Current Density & Continuity Eqn

Let there be N charges/unit volume at a point each having a charge q and moving with a mean velocity v.

The amount of charge crossing per unit area perpendicular to mean drift velocity, per unit time may be given as

$$\vec{J} = N \cdot q v \quad \text{———} (41)$$

where

$\vec{J}$ is the current density and is a vector quantity. Direction of $\vec{J}$ will depend on the velocity of the charge carriers.

Let $\vec{dS}$ be the small area element of the material. The amount of charge flowing per unit time through that area is $\vec{J} \cdot \vec{dS}$ and it corresponds to the flux of $\vec{J}$ across area $\vec{dS}$. The net charge passing through the ~~surface~~ in unit time is called electric current and is given as

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$$I = \frac{dq}{dt} = \int_s \vec{J} \cdot d\vec{s} \quad \text{——— (42)}$$

~~Also~~, consider a closed surface enclosing a volume V. If ρ is volume density of the charge total charge within d volume is

$$\int \rho \cdot dv$$

from the law of conservation of charge, the net outward flow of charge through this volume must be equal to the rate of decrease of total charge in the said volume i.e

$$\int_s \vec{J} \cdot d\vec{s} = -\frac{d}{dt}\int_v \rho \, dv \quad \text{——— (43)}$$

As the surface s is fixed, the rate of change of charge ~~of~~ within the volume is mainly due to the time rate of change of ρ. So we may write

$$\int_s \vec{J} \cdot d\vec{s} = -\int_v \frac{\partial \rho}{\partial t} dv \quad \text{——— (44)}$$

Using Gauss's divergence theorem, we can change the surface integral to volume integral as

$$\int_s \vec{J} \cdot d\vec{s} = \int_v \operatorname{div} \vec{J} \, dv = -\int_v \frac{\partial \rho}{\partial t} dv \quad \text{——— (45)}$$

(45) can also be written as

$$\int_v \left[\operatorname{div} \vec{J} + \frac{\partial \rho}{\partial t} \right] dv = 0 \quad \text{——— (46)}$$

For any arbitrary value of volume V, the integral must be zero $\Rightarrow$ the integral must be zero everywhere i.e

$$\text{div}\,\vec{J} + \frac{\partial \rho}{\partial t} = 0 \quad \text{———} \quad (47)$$

(47) is the mathematical expression of conservation of charge and is the continuity eqn.

According to the eqn:

the total current flowing out of some volume should be equal to the rate of decrease of charge within the volume. in other words it means that charge can neither be created nor be destroyed.

In case of a steady state, the charge density at any point within the region remains constant i.e

$$\frac{\partial \rho}{\partial t} = 0$$

$$\left.\begin{aligned} \therefore \quad & \text{div}\,\vec{J} = 0 \\ \text{or} \quad & \vec{\nabla}\cdot\vec{J} = 0 \end{aligned}\right\} \quad \text{———} \quad (48)$$

(48) means that there is no net outward flux of current density $\vec{J}$ i.e lines of electric current are continous.

Again experimentally, it is observed that for a given material, current density in steady state is proportional to the applied field $\vec{E}$. So we may write

$$\vec{J} \propto \vec{E}$$

$$\therefore \quad \vec{J} = \sigma \vec{E} \quad \text{———} \quad (49)$$

where σ is called electrical conductivity and is constant for a given material at a const. temp.

Reciprocal of conductivity is called resistivity, measured in ohm-metre

PHYSICAL SIGNIFICANCE OF MAXWELL'S EQNS (CORRELATION WITH ELECTRICITY AND MAGNETISM LAWS)

1st steady state eqn

1. $\vec{\nabla}\cdot\vec{D} = \rho$ —— 83

This eqn is a restatement of Gauss's law in electrostatics: Putting eqn (88) in $\vec{E}$ form

$\vec{\nabla}\cdot\vec{E} = \rho/\varepsilon$ (as $\vec{D}$ is the displacement vector given by

$\vec{D} = \varepsilon\vec{E}$ and

ρ — is the charge density and integrating over volume V, we get

$$\iiint \vec{\nabla}\cdot\vec{E}\,dv = \frac{1}{\varepsilon}\iiint \rho\,dv \quad\text{—— 84}$$

from the divergence theorem of vector analysis

$$\iiint \vec{\nabla}\cdot\vec{E}\,dv = \oiint \vec{E}\cdot d\vec{s} \quad\text{—— 85}$$

Where

$\oiint$ represents integral over a closed surface

Comparing ~~84~~ & 85 we have

$$\oiint \vec{E}\cdot d\vec{s} = \frac{1}{\varepsilon}\iiint \rho\,dv.$$

Now $\oiint \vec{E}\cdot d\vec{s}$ represents the electric flux through a closed surface and $\iiint \rho\,dv = q$ represent the charge in the closed surface

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$$\oint\!\!\oint \vec{E}\cdot d\vec{S} = q/\varepsilon \text{ ——— } (86)$$

Which means that the total flux through a closed Surface is equal to the total charge inside the surface divide by ε. This is d Gauss's Law of electrostatics.

2. $\vec{\nabla}\cdot\vec{B} = 0$ (Second steady state eqn)

where

$\vec{B}$ is d Magnetic induction vector. This eqn is d restatement of Gauss's law in Magnetism and proves that magnetic poles exist only in pairs i.e isolated magnetic poles can not be found.

For a closed Surface we have

$$\iiint \nabla\cdot\vec{B}\, dv = 0$$

Applying the divergence theorem in vector analysis we may have

$$\iiint \vec{\nabla}\cdot\vec{B} = \oint\!\!\oint \vec{B}\cdot d\vec{S}$$

$$\oint\!\!\oint \vec{B}\cdot d\vec{S} = 0 \text{ ——— } 87$$

$\oint\!\!\oint \vec{B}$ represents the magnetic flux through a closed surface, so the magnetic flux through a closed surface is zero. In other words it means that a magnetic flux entering into the vol is equal to the magnetic flux leaving the volume. So there is no source of sink in d vol, which in other words mean that isolated magnetic poles (monopoles) cannot exist.

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3. $$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = -\mu \frac{\partial \vec{H}}{\partial t} \quad \text{--- } 88$$
(1st time varying eqn)

Integrating (88) over a bounded surface bounding a path C, we get

$$\int_s (\vec{\nabla} \times \vec{E}) \cdot d\vec{s} = \oint_s \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s} \quad \text{--- } 89$$

from Stokes's theorem of vector analysis

$$\int_s (\vec{\nabla} \times \vec{E}) \cdot d\vec{s} = -\oint_c \vec{E} \cdot d\vec{l}$$

Eqn (89) can be written as

$$\oint \vec{E} \cdot d\vec{l} = -\oint_s \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s}$$

$\oint \vec{E} \cdot d\vec{l}$ represents an induced e.m.f in a closed circuit and $-\oint \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s}$ represents negative rate of change of magnetic flux through d circuit. This is Faraday's Law of electromagnetic induction. It shows that time variation of magnetic induction $\vec{B}$ generates the electric field $\vec{E}$.

4. $$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \quad \text{(Second time varying eqn)}$$

where $\vec{J}$ is the conduction current density given by
$\vec{J} = \sigma \vec{E}$

σ — being the conducting medium and
$\frac{\partial \vec{D}}{\partial t}$ — displacement current density.

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This eqn is the restatement of Ampere's Law combined together for displacement current and conduction current and states that a changing electric field produces a changing magnetic field.

Also we know that changing magnetic field produces a changing electric field. So we may conclude that: A changing electric field produces a changing magnetic field and a changing magnetic field produces a changing electric field. Alternate production of electric and magnetic fields give rise to the propagation of electromagnetic waves.

DISTINCTION BTW CONDUCTION & DISPLACEMENT CURRENT

Conduction current is the actual current that flows through the conducting mediums and so it obeys Ohm's law i.e

$I = V/R$ as well as its vector form

$\vec{J} = \sigma \vec{E}$ where

$\vec{J}$ is d conduction current density

σ — d conductivity of the medium and

$\vec{E}$ — d electric field intensity

Displacement current is the current which is set up in the dielectric medium (ie $\sigma = 0$) due to d variation of induced displacement charge produced by the changing electric field applied across the dielectric. Displacement charge density is given by $\frac{\delta \vec{D}}{\delta t} = \varepsilon \frac{\delta \vec{E}}{\delta t}$ and it has the units of current/unit area.

MAJOR CONTRIBUTION OF MAXWELL EQUS TO E-M WAVE THEORY

Maxwell introduction of displacement vector in a polarizable dielectric medium corresponding to the electric vector in vacuum or free space explained many relevant phenomenon as shown below.

Displacement vector D when related to electric field vector E is given by

$$\vec{D} = \varepsilon \vec{E}$$

where ε is the permittivity of the medium.

Now from the Maxwell's idea of displacement current, we have

$$\frac{\partial \vec{D}}{\partial t} = \frac{\partial}{\partial t}(\varepsilon \vec{E}) = \varepsilon \frac{\partial \vec{E}}{\partial t}$$

So Maxwell's second time varying eqn for e.m wave in modified form will be

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} = \sigma \vec{E} + \varepsilon \frac{\partial \vec{E}}{\partial t}$$

The term

$$\frac{\partial \vec{D}}{\partial t} = \varepsilon \frac{\partial \vec{E}}{\partial t}$$ accounts for all radiative

processes from a charge (positive or negative) undergoing an acceleration. Various ways for this to happen may be when an electron:

(i) Jumps from high energy level to low energy level due to the de-excitation of atom.

(ii) Suffers collision and its path changes and so gets accelerated.

(iii) Undergoes oscillatory motion in an alternating field as in the case of a wireless antenna.

DIFFRACTION

The phenomenon of bending light round the corners and spreading of light waves into the geometrical shadow of an obstacle placed in the path of light is called diffraction.

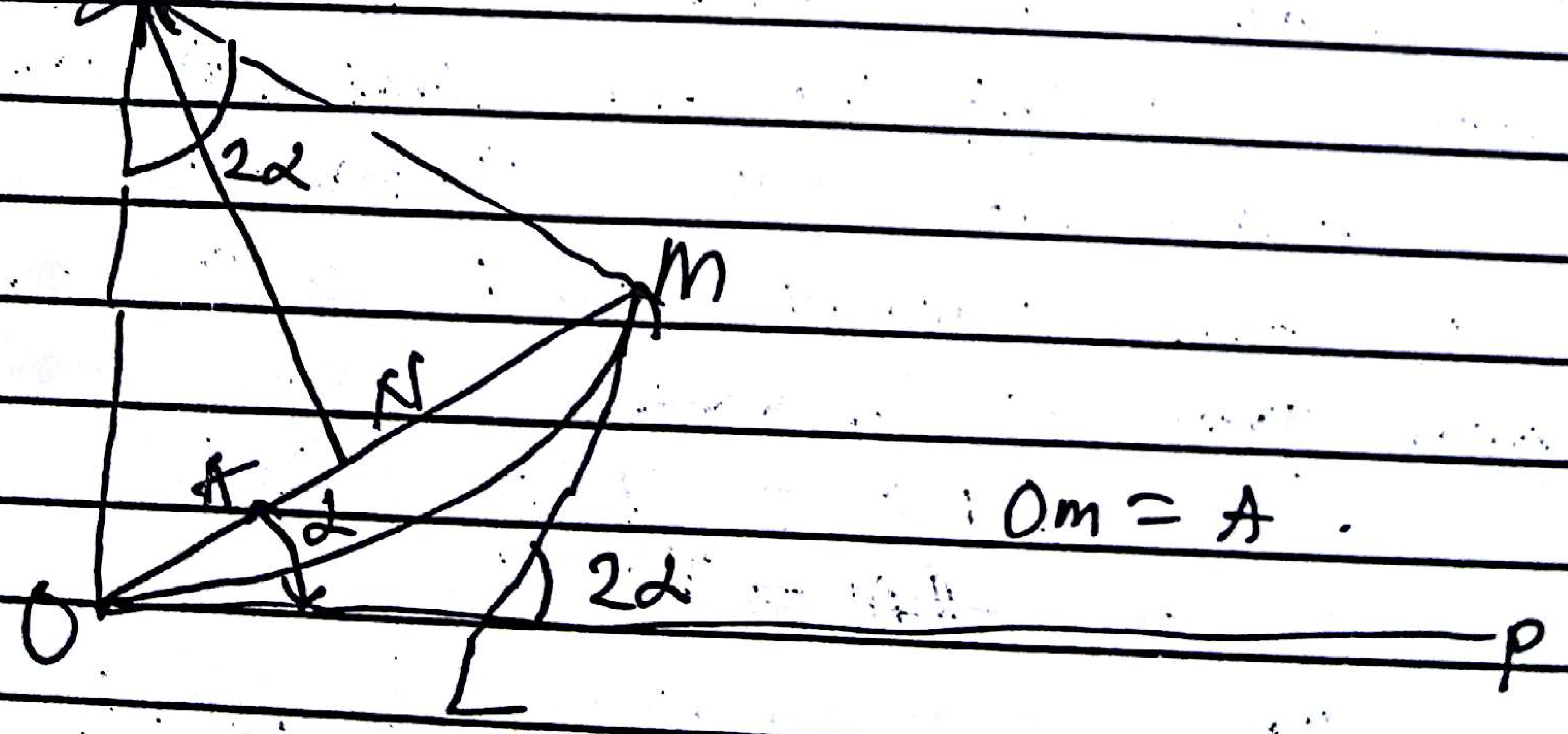
Basically, the diffraction phenomenon is divided into two main groups:

(i) Fraunhofer Diffraction (ii) Fresnel Diffraction.

(i) Fraunhofer Diffraction

It consists of all those cases in which either the source or screen or both are at infinite distance from the obstacle. This may be achieved by using two convex lenses, one to make the light from the source parallel before it falls on the aperture and the other to focus the light after diffraction on the screen.

Intensity distribution in the Diffraction Pattern due to a single slit



$OM = A$.

We can explain the intensity variation in the diffraction pattern due to a single slit as follows:

The incident wavefront on the slit AB (fig above) can be

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Imagined to be divided into a large number of infinitesimally small strips. The path difference b/w the secondary waves originating from the extreme points A & B is

$a \sin\theta$

where a — is the width of the slit and

$\angle ABN = \theta$

For a parallel beam of incident light, the amplitude of vibration of the waves from each strip can be taken to be the same.

If we consider the secondary waves in a direction inclined at angle θ from the point B upwards, with change of path difference, phase diff also increases.

Let α be the phase diff b/w the secondary waves from the point B & A of the slit (fig above)

Since we have divided the wavefront into a large number of strips, the resultant amplitude due to all the individual small strips can be gotten by the vector polygon method. In case, the amplitudes are small and the phase diff increases by infinitesimally small amount from strip to strip. So the vibration polygon coincides with the circular ~~arc~~ is OM. OP gives the direction due to the secondary waves from A. R is the centre of the circular. From geometry we have

$$\angle MLP = 2\alpha$$

$$\angle ORM = 2\alpha$$

In ΔORN

$$\sin\alpha = \frac{ON}{r}, \quad ON = r\sin\alpha$$

Where r is the radius of the circular arc:

$$\text{chord } OM = 2ON = 2r\sin\alpha \qquad (1)$$

The length of d arc OM is proportional to the width of d slit

∴ length of d arc ~~OA~~

$$OM = K\alpha \quad (K - \text{proportionality const})$$

or

$$2r = \frac{K\alpha}{\alpha} \quad \text{—— (2)}$$

Putting this value of 2r in eq (1), we get

$$\text{Chord } OM = \frac{K\alpha}{\alpha} \quad \text{—— (3)}$$

Also,

$OM = A$ (is d amplitude of d resultant), so we have

$$A = (K\alpha)\frac{\sin\alpha}{\alpha}$$

$$A = A_0 \frac{\sin\alpha}{\alpha} \quad \text{—— (4)}$$

Which means that the resultant amplitude of vibration at a point on d screen is given by

$$I = A^2 = A_0^2 \frac{\sin^2\alpha}{\alpha^2} = I_0\left(\frac{\sin\alpha}{\alpha}\right)^2 \quad \text{—— (5)}$$

i.e intensity at any point on the screen is proportional to $\left(\sin\alpha/\alpha\right)^2$

Now phase difference of 2π corresponds to a path diff of λ. So as phase diff of ~~2d~~ 2α will be given by

$$2\alpha = \frac{2\pi}{\lambda} \cdot a\sin\theta$$

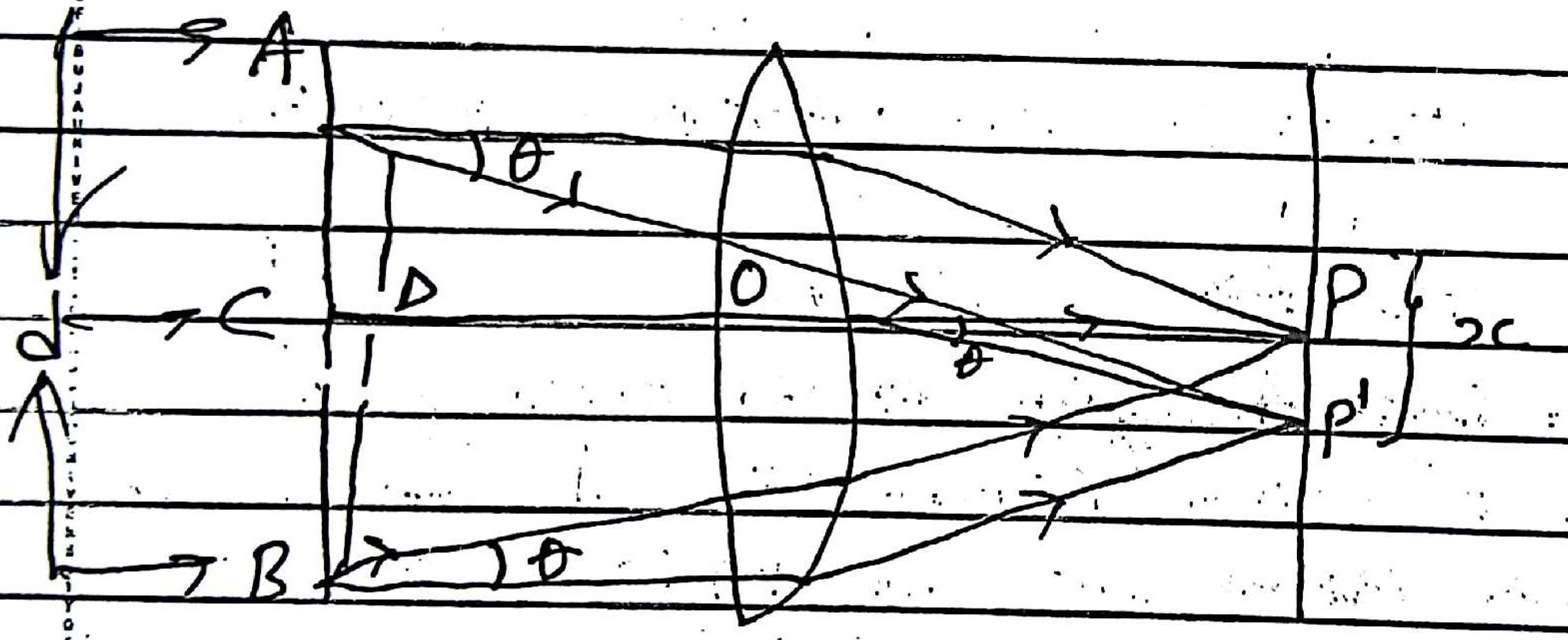
Where $a\sin\theta$ is d path diff btw the secondary waves from A & B (fig above) or

$$\alpha = \frac{\pi}{\lambda} a \sin\theta \quad \text{---} \quad (b)$$

It means that α depends on d angle of diffraction θ. Also $\frac{\sin^2\alpha}{\alpha^2}$ will give intensity at different points depending upon d value of θ.

In d fig above we have d intensity distribution curve for α or $\sin\theta$ along x-axis v/s $\frac{\sin^2\alpha}{\alpha^2}$ along y-axis.

Fraunhofer Diffraction at a Circular Aperture



From the fig above AB is a circular aperture of diameter d. Let C be the centre of aperture and P a point on the screen. OP is ⊥ar to d screen and d screen is ⊥ar to d plane of d paper.

Let us consider a plane wavefront incident on the circular aperture. The secondary waves travelling in the direction CO come to focus at P. So P corresponds to d position of the central maximum. Here, all d secondary waves coming from points which are at d same distance from O reach P to reinforce each other.

If we consider secondary waves moving in a direction inclined at an angle θ with CP, they will be reaching at point P is such that $PP' = x$

Now the path diff btw the waves coming from pts A & B (i.e from extreme positions) is AD, where

$$AD = d\sin\theta \quad \text{——— (7)}$$

Now we will have the pt P to be centre of minima if path diff is equal to integral multiple of λ i.e

$$d\sin\theta = n\lambda \quad \text{——— (8)}$$

and for P' to be centre of maxima path diff must be odd multiple of $\lambda/2$ i.e

$$d\sin\theta = (2n+1)\lambda/2 \quad \text{——— (9)}$$

If P is at minimum intensity point, then all the points which are at equal distance from P and P' lying on a circle with radius x will be positions of minima.

Again, if we place the converging lens very near to the slit, or when the distance of the screen is very large from the lens

$$\sin\theta = \theta = \frac{x}{F} \quad \text{——— (10)}$$

Also, for the 1st secondary minima, we have

$$d\sin\theta = \lambda$$

$$\sin\theta = \theta = \frac{\lambda}{d} \quad \text{——— 11}$$

From eqs (10) & 11

$$\frac{x}{F} = \frac{\lambda}{d}$$

or

$$x = F\lambda/d \quad \text{——— 12}$$

where x is the radius of the Airy's disc. But experimentally the radius of the 1st dark ring is slightly more given by eqn (12), which according to Airy is given as

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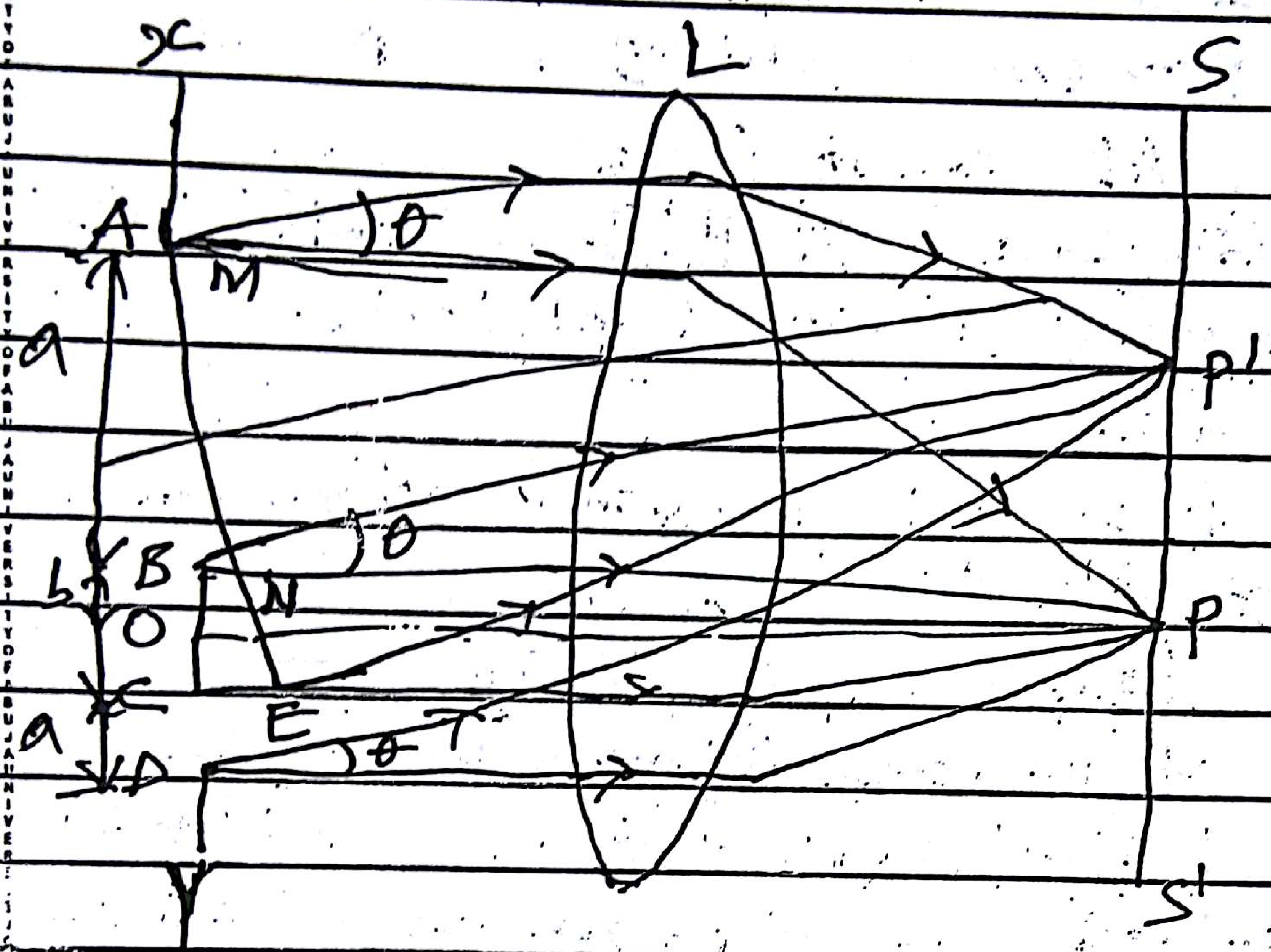
$x = 1.22 f\lambda / d$ ——— 13

Fraunhofer Diffraction at a Double slit



As shown in the fig above, AB & CD are two rectangular slits with each having width a and b as d width of the opaque portion. Both d slits are parallel to one another and $\perp$ar to d plane of paper. Let L be d collimating lens and SS' be $\perp$ screen to d plane of paper. Let P be a point on d screen such that OP is $\perp$ar to both screen and d slits.

Now let a plane wavefront XY be incident so that all secondary waves moving along parallel direction to OP come to focus at O i.e P is d position of central maxima.

We have to consider d diffraction pattern constituted by the following parts:

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(i) Interference fringes due to secondary waves from the two slits taken separately.

(ii) Diffraction fringes due to the secondary waves from the two slits taken separately.

Let us denote the angle of diffraction θ, for calculating the interference pattern and angle ϕ for calculating the diffraction pattern.

Both θ & ϕ refer to the direction of secondary waves of direction of incident light.

(i) Interference Maxima & Minima.

Let the secondary waves travelling in a direction inclined at an angle θ with the initial direction, then we have

In ΔACN

$$\sin\theta = \frac{CN}{AC} = \frac{CN}{a+b}$$

or $$CN = (a+b)\sin\theta \qquad (14)$$

Now if this path diff is equal to odd multiples of $\lambda/2$, θ gives the direction of minima due to interference of the secondary waves from the two slits

$$CN = (a+b)\sin\theta_n = (2n+1)\lambda/2 \qquad (15)$$

for $n = 1, 2, 3$ etc. the values of $\theta_1', \theta_2', \theta_3'$ etc. corresponding to the directions of minima can be known

Again from eqn (15)

$$\sin\theta_n = \frac{(2n+1)\lambda}{2(a+b)} \qquad (16)$$

Again, if the secondary waves travel in a direction θ' such that the path difference is even multiples of $\lambda/2$. The θ' gives the direction of the maxima due to interference of light waves coming from the two slits.

$$CN = (a+b)\sin\theta_n' = 2n \cdot \lambda/2$$

$$\sin\theta_n' = \frac{n\lambda}{(a+b)} \quad \text{———} \quad (17)$$

For $n = 1, 2, 3$ etc the values of $\theta_1', \theta_2', \theta_3'$ etc. corresponding to the directions of minima can be known from eqn (17)

$$\sin\theta_1 = 3\lambda/2(a+b)$$

and

$$\sin\theta_2 = 5\lambda/2(a+b)$$

$$\sin\theta_2 - \sin\theta_1 = \lambda/a+b \quad \text{———} \quad (18)$$

Which means that angular separation b/w any two consecutive maxima or minima is $\lambda/(a+b)$ it means that angular separation $\propto \lambda/(a+b)$

where

$(a+b)$ is the distance between the two slits.

(ii) Diffraction Maxima & Minima — Consider the secondary waves travelling along a direction at an angle ϕ with the initial direction of incident light.

In this case if the path diff BM is equal to λ, the wavelength of light used then the angle of diffraction will be ϕ for getting diffraction minima.

It means that ~~the~~ path diff b/w the secondary waves originating from the points A & B i.e. extremeties of a slit

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will be equal to λ. If we consider the wavefront on AB to be made up of two halves, then $\lambda/2$ will be the path difference btw corresponding points of upper and lower half of the wavefront.

So the effect at P' due to these two halves of the wavefront will be zero. Similarly, the effect at P' for same direction of secondary waves, due to wavefront incident on the slit CD is also zero.

$$a \sin\theta_n = n\lambda$$

when

$n = 1, 2, 3, --$ we get the corresponding order of diffraction pattern.

MISSING ORDER IN A DOUBLE DIFFRACTION PATTERN

We have taken the slit width as a and double slit separation as b. If a is kept const, we will observe the same diffraction pattern. However, if a is kept constant and b is varied, the space btw interference maxima changes, depending upon the relative values of a and b. Some order of interference maxima will be missing in the resultant pattern.

We have direction for interference maxima as

$$(a+b) \sin\theta = n\lambda \quad \text{——— } 20$$

and the direction for diffraction minima as

$$a \sin\theta = m\lambda \quad \text{——— } 21$$

where

n & m are integers.

If the values of a & b are such that both ~~the~~ eqs 20 & 21 are satisfied simultaneously, for same value of θ, in that case position of interference maxima correspond to that of diffraction minima.

Let us consider some cases

(i) If $a = b$

$$2a\sin\theta = n\lambda \quad \text{——— (22)}$$

and $$a\sin\theta = m\lambda \quad \text{——— (23)}$$

$\therefore$ from eqⁿ (22) & (23)

$$\frac{n}{m} = 2$$

$$n = 2m$$

if $m = 1, 2, 3 - - -$

$\therefore n = 2, 4, 6, - - -$

i.e 2nd, 4th, 6th etc, orders of the interference maxima will be missing in the diffraction pattern.

(ii) if $2a = b$

$$3a\sin\theta = n\lambda$$

and $$a\sin\theta = m\lambda$$

$$\therefore \frac{n}{m} = 3$$

or

$$n = 3m$$

putting $m = 1, 2, 3, —$ etc

$\therefore n = 3, 6, 9$ — etc

So ~~the~~ 3, 6, 9, etc. of ~~the~~ interference maxima will be missing in the diffraction pattern.

(iii) if

$$a + b = a \quad \text{i.e } b = 0$$

In ~~this~~ case we will have a single slit

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So all interference pattern will be missing. In the case diffraction pattern observed on the screen is similar to that due to single slit with width 2a.

FRESNEL INTEGRAL

For any point of Cornu's spiral (the effect at a point due to an incident wavefront) we introduce two integrals for x & y coordinates, which are called Fresnel integrals. Let us consider a point P on the spiral which has a distance v from the origin. Let the length to the curve of P make an angle δ with X-axis, where δ will correspond to the phase change from O to P. Let the change in co-ordinates of the point be dx & dy for a small displacement dv of the point along the curve.

$$dx = dv \cos\delta \quad \text{--- } 25$$

and

$$dy = dv \sin\delta \quad \text{--- } 26$$

putting

$\delta = \frac{\pi}{2}v^2$ we get

$$dx = \cos\left[\frac{\pi v^2}{2}\right] dv \quad \text{--- } 27$$

and

$$dy = \sin\left[\frac{\pi v^2}{2}\right] dv \quad \text{--- } 28$$

The coordinates x and y of the Cornu's spiral are ~~thereby~~ given as

$$x = \int dx = \int_0 \cos\left[\frac{\pi v^2}{2}\right] dv \quad \text{--- } 29$$

$$y = \int dy = \int_0 \sin\left[\frac{\pi v^2}{2}\right] dv \quad \text{--- } 30$$

29 & 30 are called Fresnel's integral.

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Plane Diffraction Grating

An arrangement which is equivalent in action to a number of parallel, equidistant, small rectangular slits of equal width ~~placed~~ is called diffraction grating. Since the slits are arranged in a plane, so diffraction grating is called plane diffraction grating and is a very good ~~device~~ & accurate device for the study of spectra.

In diffraction grating, ~~the~~ slits are very narrow and are separated by opaque spaces, which do not allow light to pass ~~through them and they are called~~ "opacities". Between ~~the~~ opacities, we ~~do not allow light~~ have ~~the~~ slits which allow ~~the~~ light to pass through ~~and they~~ are called "transparencies".

Gratings are made by ruling equidistant parallel lines on an optically transparent sheet of material, the lines ~~pattern~~ being opaque and ~~the~~ space b/w the lines will be transparent to light. In the construction of a good quality grating, the following considerations must be taken into account:-

(i) it should have large number of slits i.e the number of lines should be very large. Normally ~~this~~ number may be b/w 12,000 to 30,000 lines/inch.

(ii) The spacings b/w ~~the~~ lines should be equal.

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Dispersive Power of Grating

The change in angle of diffraction corresponding to a unit change in wavelength of light used, measures the dispersive power of the grating.

from the condition of the nth order maxima, we have

$$(a+b)\sin\theta = n\lambda \quad \text{--- } 31$$

which shows that for a given order n, and grating element $(a+b)$, θ changes with λ. If wavelength changes from λ to $\lambda + d\lambda$, the corresponding angle of diffraction will change from θ + $(\theta + d\theta)$. The factor $\frac{d\theta}{d\lambda}$ is called dispersive power of the grating and $\frac{d\theta}{d\lambda}$ may be gotten by differentiation of eq (31)

$$(a+b)\cos\theta \frac{d\theta}{d\lambda} = n$$

$$\frac{d\theta}{d\lambda} = \frac{n}{(a+b)\cos\theta} = \frac{1}{(a+b)} \times \frac{1}{\cos\theta}$$

or

$$\frac{d\theta}{d\lambda} = \frac{m \times n}{\cos\theta}$$

where

$m = \frac{1}{a+b}$ number of lines/cm of length.

for small values of θ, $\cos\theta$ is practically constant, so that $d\theta$ is directly proportional to $d\lambda$. Such a spectrum is called Normal spectrum. it is also clear from above that dispersive power varies directly with order of spectrum n and number of lines/unit length m.

Resolving Power of a Diffraction Grating

The resolving power of a grating is its ability to show two neighbouring lines in a spectrum as separate.

It may also be defined as ~~the~~ wavelength of any spectral line to its difference of wavelengths b/w this line and neighbouring line such that the two spectral lines can be seen as separate.

Two lines of wavelengths λ and $\lambda + d\lambda$ are said to be resolved if the central maxima due to $\lambda + d\lambda$ falls on the first minima of λ.

The resolving power of diffraction grating is given by $\lambda / d\lambda$.

Consider two wavelets of wavelength λ and $\lambda + d\lambda$ to be incident normally on the surface of the grating. The two wavelengths will give their own diffraction patterns. According to Rayleigh's criteria, the two patterns would be ~~best~~ resolved if the principal maxima of one falls on the first secondary minima of the other in any order.

The condition for the 1st secondary minimum after the nth order maximum is given by:

$$(a+b)\sin(\theta + d\theta) = n\lambda + \lambda/N \quad \text{---} \; 32$$

Let the nth order principal maximum corresponding to wavelength $\lambda + d\lambda$ has the same direction as that of the 1st secondary minimum of the nth order principal maximum corresponding to wavelength λ, then, Candidate's Number.......................................

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$(a+b)\sin(\theta+d\theta) = n(\lambda+d\lambda)$ —— (33)

from eqns (32) & (33)

$$n\lambda + \frac{\lambda}{N} = n\lambda + n\,d\lambda$$

$$n\,d\lambda = \frac{\lambda}{N}$$ —— (34)

or

$$\lambda/d\lambda = N \times n = $$ (Total no of lines on grating × order of spectrum)

PRODUCTION OF PLANE POLARIZED LIGHT

Some methods for producing plane polarized light include:

(i) Reflection

(ii) Transmission through a pile of plates

(iii) Double refraction

(iv) Selective absorption in crystals.

(v) Scattering

For the purpose of this course we will consider double refraction.

DOUBLE REFRACTION (BIREFRINGENCE)

When a beam of ordinary light is allowed to pass through a calcite or quartz crystal, we get two refracted beams instead of the usual one as in the case of glass. This phenomenon is called double refraction or birefringence.

To show the significance of the phenomenon, allow a narrow beam of light from a point source to pass through a crystal of iceland spar or calcite, we see two images on the screen as shown in the fig below

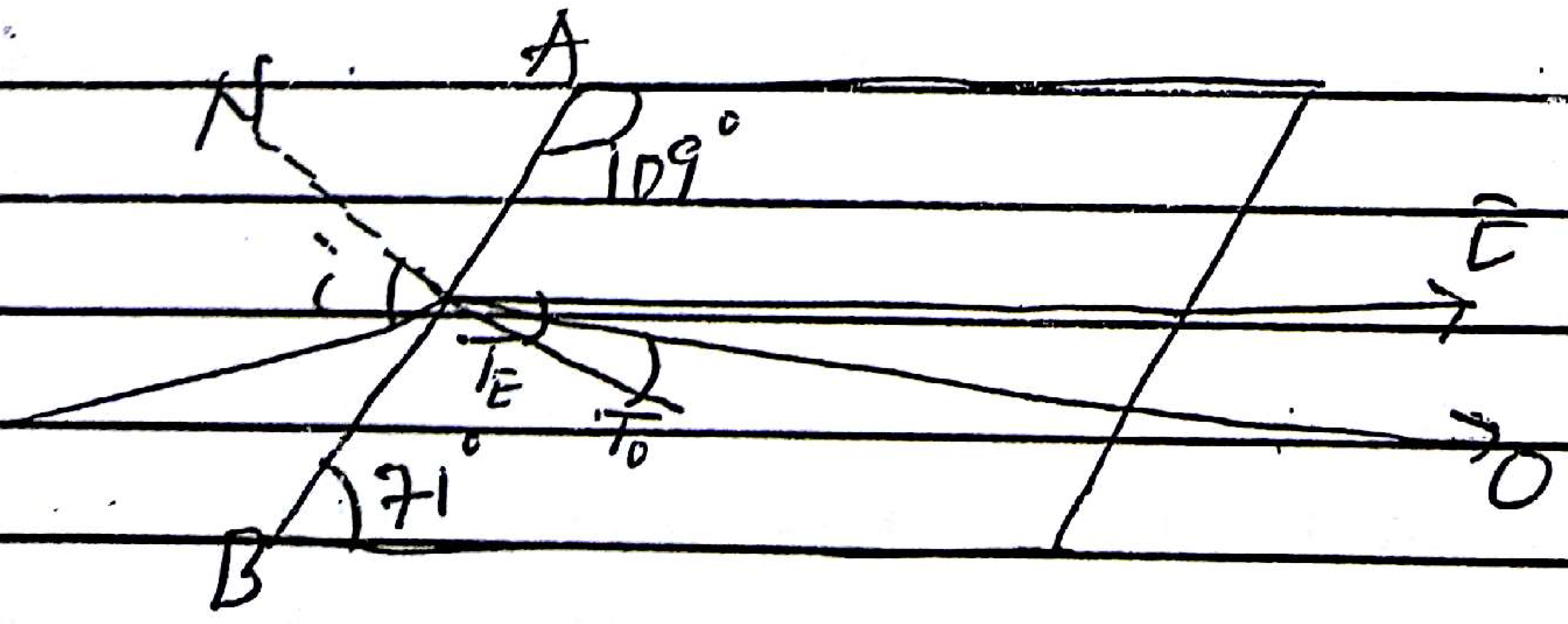

Fig A

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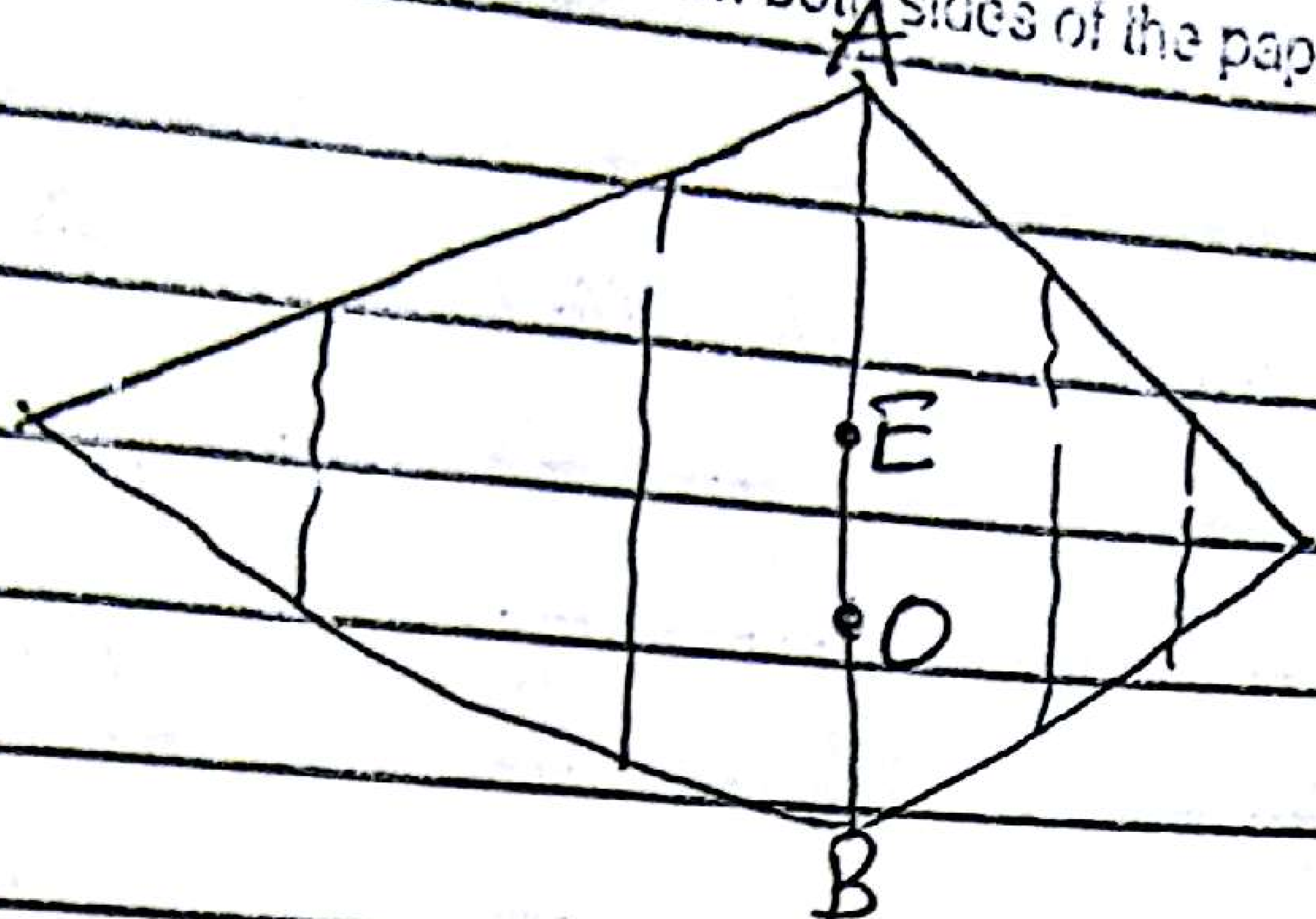

The above phenomenon can be observed in a simple way by putting a dot on a paper and seeing it through a calcite crystal when two images will be observed. If the crystal is rotated one image remains stationary while the other rotates round the first. The stationary image is called ordinary image and the refracted ray which produces it is called ordinary ray and these rays obey the laws of refraction. The other image is known as extraordinary image and the refracted rays producing this image are called extraordinary rays because they do not obey the laws of refraction.

Normally the two opposite faces of a calcite crystal are always parallel so that both the refracted rays, i.e., ordinary and extraordinary rays emerge parallel to the incident beam and hence parallel to each other. For normal incidence the ordinary ray will pass straight through without deviation whereas the extraordinary ray will be transmitted at some angle and come out parallel to and displaced from the incident ray. As shown above on rotating the crystal about the beam as axis, the E ray rotates about the fixed O ray, the line joining the shorter diagonal of the

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emergent crystal face.

It may further be noticed that both ordinary and extraordinary rays are plane ~~pla~~ polarized i.e. their planes of polarization are ⊥ar to each other.

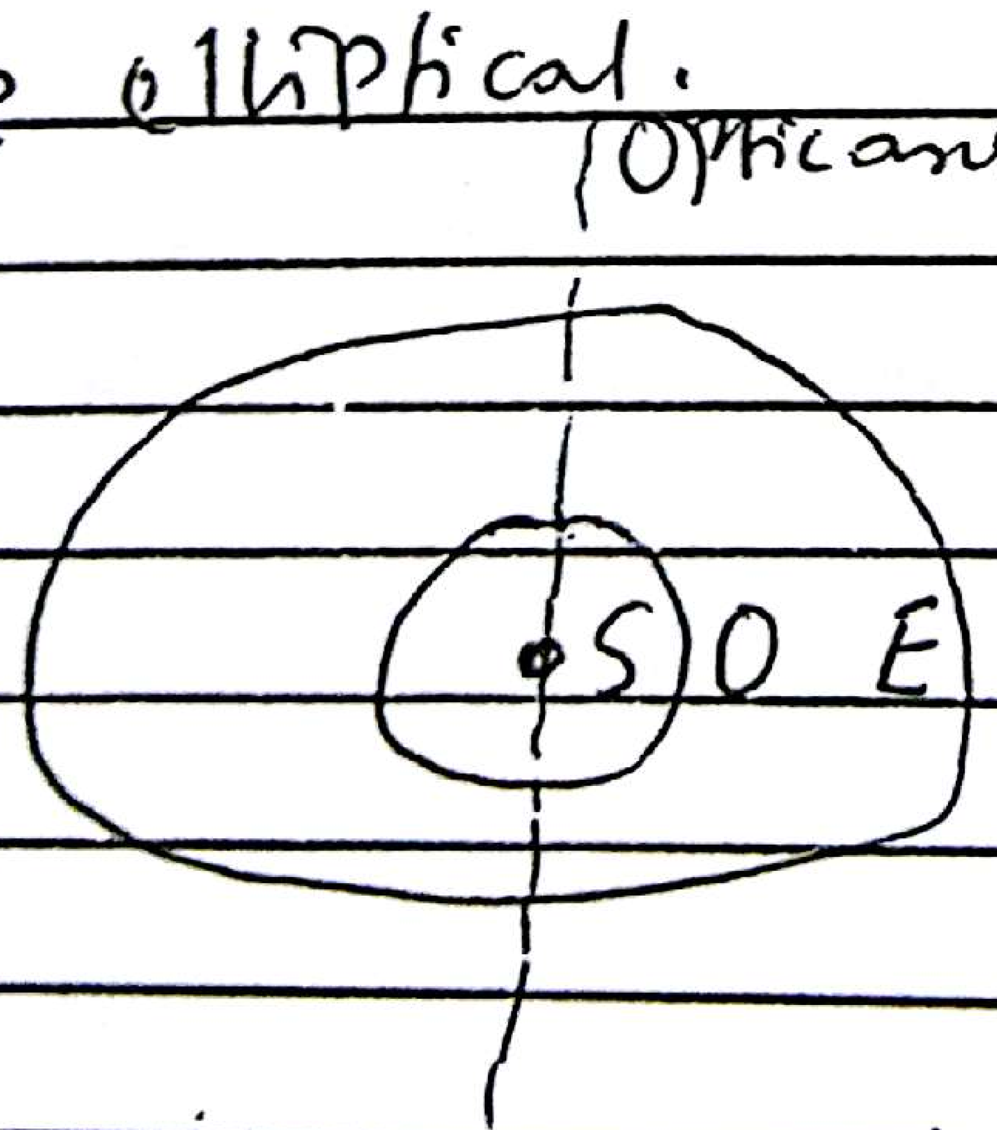
HUYGEN'S THEORY OF DOUBLE REFRACTION

Huygen's extended his theory of secondary wavefronts to explain the phenomenon of double refraction in uniaxial crystals. According to this theory:

(i) A point source of mono-chromatic light in a double ~~re~~ refracting medium is the origin of two wavefronts.

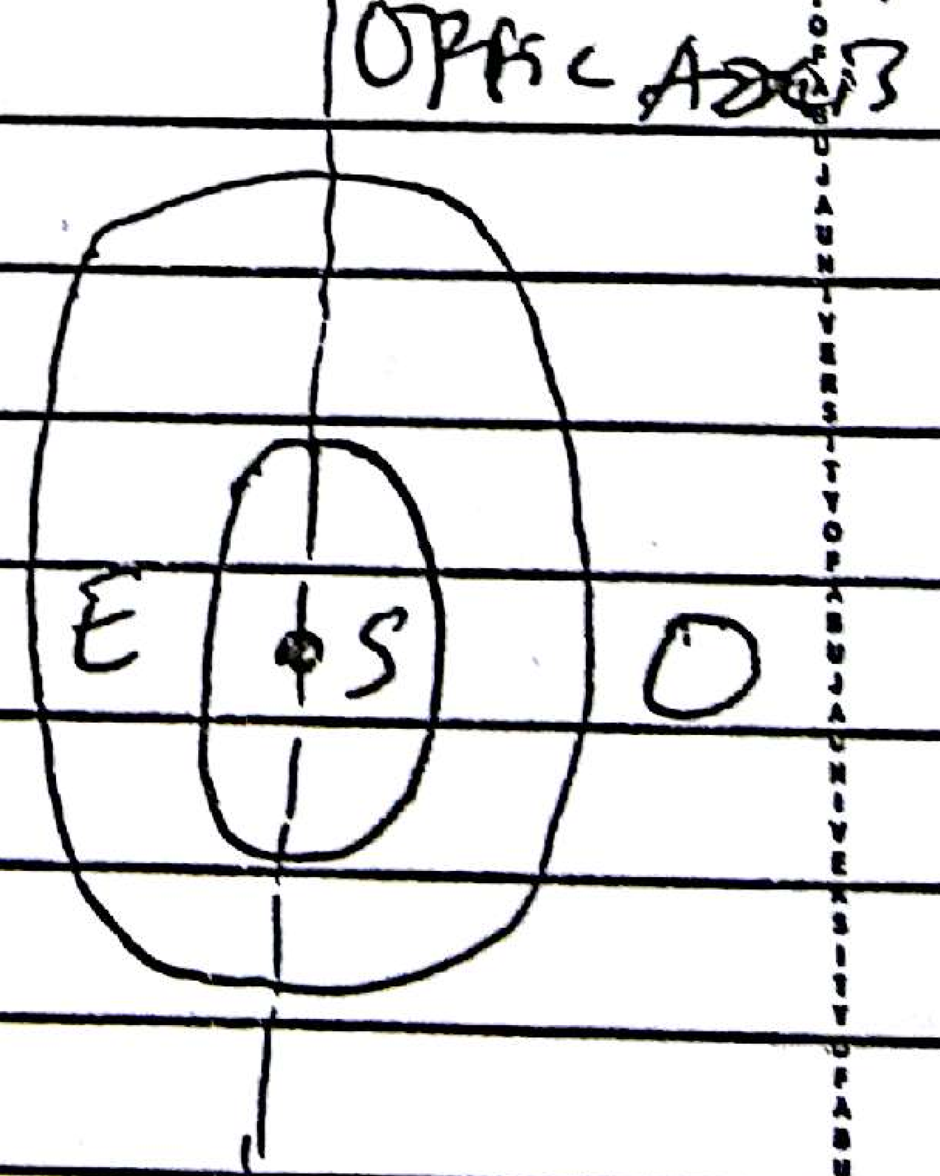
(ii) The ordinary wave travels with same velocity in all directions and so the corresponding wavefront will be spherical.

(iii) Extraordinary waves have different velocities in different directions, so the corresponding wavefront will be elliptical.



Negative Crystal

(C)



Positive Crystal

(D)

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Since along ~~the~~ optic axis both O-ray and E-ray travel with the same velocity, the two wavefronts touch each other at two points. The line joining these points will give the direction of optic axis as shown in fig (c). In general, Sin both O-ray and E-ray travel with different velocities in different directions, these rays separate out in passing through such crystals and double refraction takes place.

NEGATIVE CRYSTALS

The wavefronts surrounding a point source of light S in such crystals is given in fig (c). The fig shows a section in the plane of paper and sphere which gives the wave surface for O-ray and ellipse for E-ray. It is evident that

(i) O-ray surface lies inside E-ray surface.

(ii) The velocity of O-ray is constant in all directions.

(iii) The velocity of E-ray varies with direction. It is minimum along the direction of optic axis.

(iv) Refractive index for E-ray is less ~~than~~ ~~that~~ of O-ray ($\mu_o < \mu_e$).

This type of crystals are called -ve crystals e.g. Calcite is a -ve crystal.

POSITIVE CRYSTALS

The wavefronts surrounding a point source S in such a crystal are shown in fig (d). It is evident that

(i) O-ray surface lies outside E-ray surface.

(ii) Velocity of O-ray is constant in all directions.

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(iii) The velocity of E-ray varies with direction. It is maximum along optic axis where it will be equal to velocity of O-ray ~~maximum~~ and minimum when it is ⊥ar to the direction of optic axis

(iv) Refractive index for E-ray is more than that for O-ray ($\mu_e > \mu_o$). Such crystals are called positive crystals e.g. quartz is a +ive crystal.

FARADAY EFFECT

Faraday effect was discovered in 1845 and corresponds to the effect of a strong magnetic field on an isotropic substance of high refractive index. It is observed that if an isotropic substance of high refractive index is placed in a strong magnetic field it becomes temporarily an optical active substance. In case a plane polarized light is passed through such a substance such that the direction of propagation is along the direction of the field, then it is observed that the plane of vibration gets rotated through certain angle θ which depends on the strength of magnetic field H and the length L of the path of light through the substance. This can be written as

$$\theta = V \cdot H \cdot L \quad \text{——— (1)}$$

where V is called Verdet Constant and it dependent on nature of substance. This effect called Faraday Effect is shown by substances like carbon bisulphide, glass and water. It is experimentally observed that rotation angle is ~~doubled~~ doubled, if the light after passing through the temporarily optical active substance

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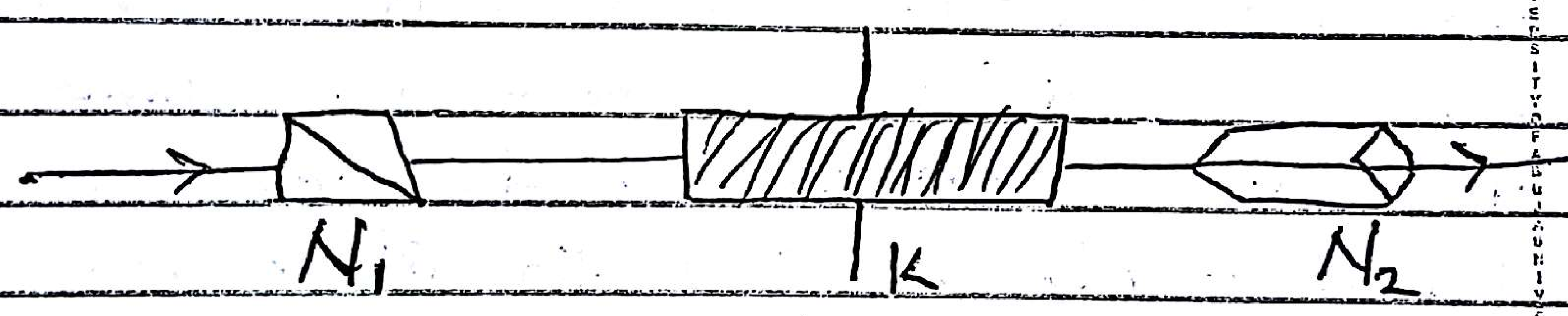
is reflected back in the opposite direction.
In case of ordinary optical ~~device~~ active substance like quartz, calcite etc., the effect is annulled when light is sent back after reflection through them.

KERR EFFECT

Kerr in 1875 discovered experimentally the effect of strong electric field on some transparent substances like Nitrobenzene. It was observed that the electric field rotated the plane of polarization of light when the transparent substance is placed b/w a strong electric field. The effect has been given ~~the~~ name Kerr effect.



The fig above shows a Kerr cell which is a small container with two electrodes immersed in the transparent insulating liquid (ortho-Dichlorobenzene or Nitro Benzene). A very high p.d is applied b/w the two electrodes. The cell behaves like a uniaxial crystal with optic axis parallel to the line of force of the electric field. Cell is placed b/w two crossed Nicols N_1 & N_2.
If a plane polarized light from N_1 is made to pass through Kerr cell, it split into two components. One component is parallel to the electric field while the other is ⊥ar to the electric field. The two ~~co~~ components move with different velocities inside the cell

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and hence some phase difference is introduced b/w them.

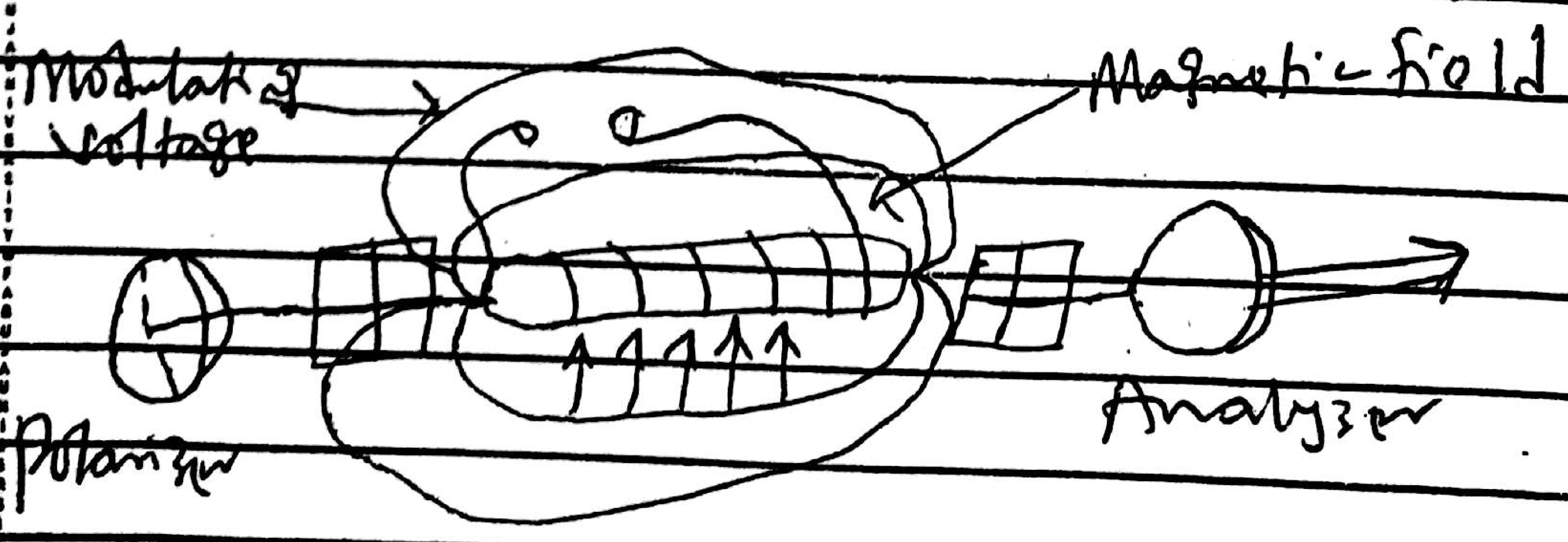
It is observed that the emergent light is elliptically polarized which now enters the analyzing Nicol N_2. Again the light is broken up into two components at N_2: One parallel and the other per. to the principal section of N_2. per component is lost in the Nicol prism N_2, due to internal reflections while the other emerges through N_2.

Therefore, when a high voltage is applied to the electrodes of Kerr cell, light is transmitted through the system. However, as the electric field is switched off, no light is available after the Nicol N_2, as the two Nicols are crossed. Thus testifying the effect of electric field in creating optical activity in certain transparent substances.

COTTON-MOUTON EFFECT

This effect is magneto-optic effect.

It is observed that certain isotropic materials acquire the optical behavior of a uniaxial crystal under an applied external magnetic field.



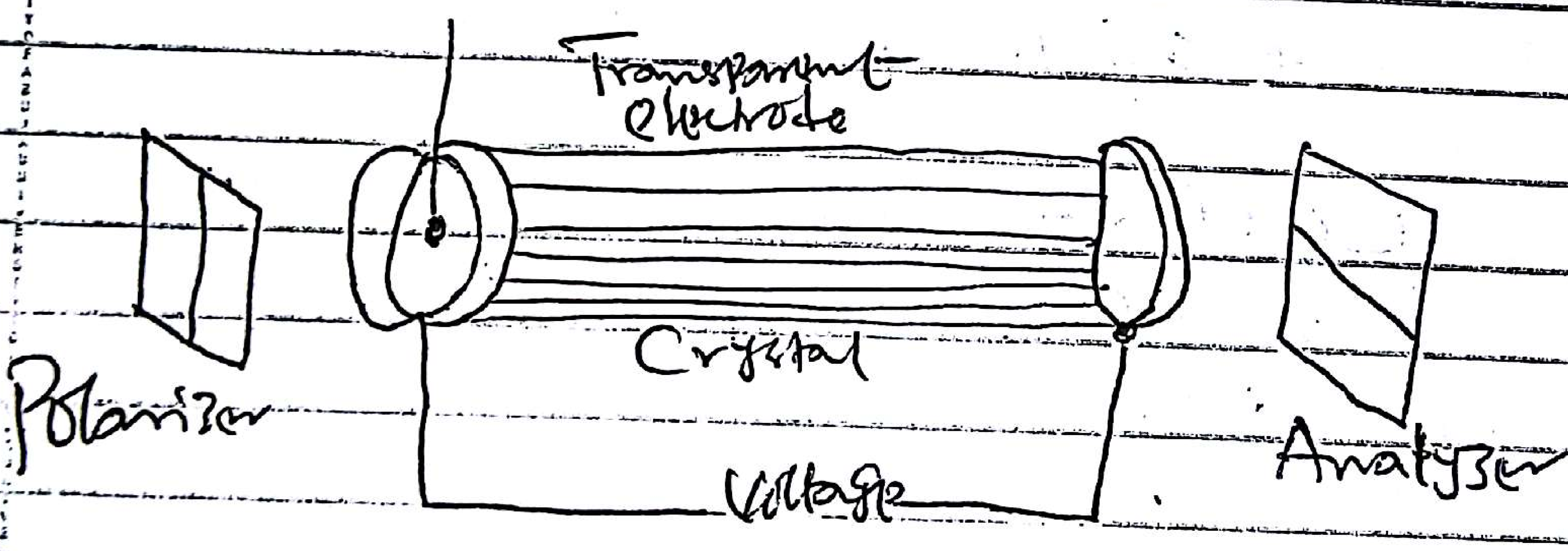
The induced birefringence is given by the relation

$$\Delta\mu = C.\lambda.B$$

The magnitude of induced birefringence is found to be very small. In the above relation C is a constant, B is d applied magnetic field and λ is the wavelength of light.

POCKETS EFFECT

Pocket effect is an opto-electric effect.
F. Pockets in 1883 found that when an electric field is applied to some piezo-electric crystals, birefringences are produced.



Normally piezo-electric crystals are birefringents but birefringence is restricted to only some preferred directions. However, if an external electric field is applied on the piezo-electric crystals, birefringence (double refraction) is observed in other directions as well.
A Pocket cell consists of piezo-electric crystal with suitable orientation such that the transparent electrodes of tinoxide or indium are put on the opposite

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faces.

The birefringence induced in the crystal is proportional to the strength of the applied electric field. i.e.

$$\Delta \mu \propto E$$

$$\Delta \mu = KE$$

Where K is a const which depends on the nature of the material.

A Pockel cell is simple in construction and requires very small applied voltage $\simeq 1.5KV$ unlike Kerr cell which requires very high voltage $\simeq 15KV$. Piezo-electric crystals like Potassium-Di-hydrophosphate (KDP) and Ammonium-Di-hydrophosphate (ADP) are extensively used as Pockels cells.

Because of their optoelectric properties Pockels and Kerr cells are widely used as optoelectric shutters in Q-switching in Lasers.