In time $dt$ (ie $t+dt-t$) the radius vector ~~$\vec{r} = OA$ sweeps~~

$$\vec{r} = \vec{OA}$$

sweeps the area (shaded) OAB which may be considered as a triangle determined by the positions vector $\vec{r}$ and the displacement $d\vec{r}$ where

$$d\vec{r} = \vec{AB}$$

The area OAB can be represented by a vector

$$d\vec{A} = \frac{1}{2}\vec{r} \times d\vec{r}$$

$$\frac{d\vec{A}}{dt} = \frac{1}{2}\vec{r} \times \frac{d\vec{r}}{dt} \qquad v = r\omega \qquad \omega = \frac{\theta}{t} = \dot{\theta}$$

$$= \frac{1}{2} r \times v \qquad \text{ie} \qquad \therefore v = r\dot{\theta}$$

$$= \frac{1}{2} r^2 \dot{\theta}$$

The laws of area will hold iff

$$\frac{d\vec{A}}{dt} = \text{constant} \quad \text{or} \quad \vec{r} \times \vec{v} = \text{constant}.$$

3. The squares of the periods of revolution are proportional to the cubes of the average distances of the planets from the sun.

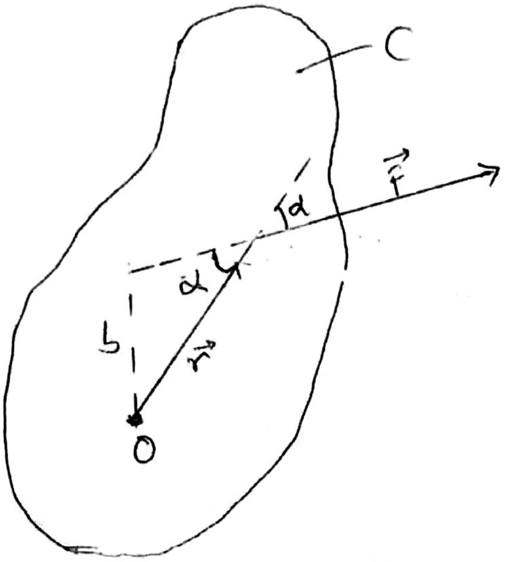
$$T^2 = K r_{av}^3 \qquad r_{av} := \text{average distances}$$

Remark

1. Modern observations on the solar system, on the moon of Jupiter and on earth's satellites confirm the success of the three laws as summaries of the data.
2. The three laws can only be accounted for if the laws of force which applies to the sun and the planet is an inverse square law – the vital one being the first law. This is because only an inverse square can lead to planetary ~~motion~~ orbits which are elliptical with the sun at one focus.
3. The second law is valid for any centrally directed force.

FORCES ACTING ON A RIGID BODY

When a force acts on an extended body the body does not merely move in the direction of the force but usually changes its orientation by turning. Consider a force F acting on a body C suppose that the effect of the force is to rotate the body around O.



Torque, $\tau \equiv$ moment of force

from the diagram

$$\frac{b}{r} = \sin\alpha$$

or

$$b = r\sin\alpha \quad (28)$$

$$\tau = r\sin\alpha\, F$$

or

$$\vec{\tau} = \vec{r} \times \vec{F} \quad (29)$$

If the forces are congruent ie all are applied at the same point, then the resultant force, F

$$F = F_1 + F_2 + \cdots + F_n$$

$$= \sum F_i \quad (30)$$

If several concurrent forces act at a point A, the torque of each force related to each point the $\tau_i$

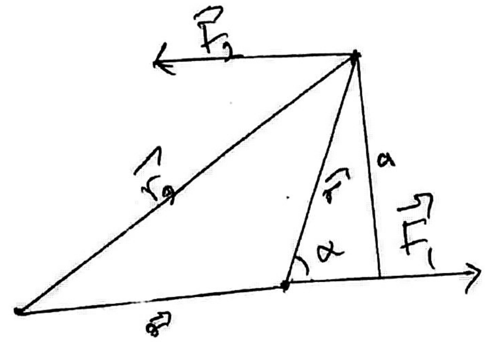
$$\vec{\tau}_i = \vec{r} \times \sum F_i$$

$$\vec{\tau} = \vec{r}\,(\vec{F}_1 + \vec{F}_2 + \cdots F_n)$$

$$= \vec{\tau}_1 + \vec{\tau}_2 + \cdots + \tau_n = \sum \vec{\tau}_i \quad (31)$$

## COUPLE OF FORCES

A couple is defined as a system of two forces of equal magnitude but in opposite direction acting along parallel lines




$$\vec{\tau} = \vec{\tau}_1 - \tau_2$$

$$= \vec{r}_1 \times \vec{F}_1 + (\vec{r}_2 \times \vec{F}_2)$$

but

$$\vec{F}_1 = -\vec{F}_2$$

$$F_2 = -\vec{F}_1$$

$$\vec{\tau}_1 = \vec{r}_1 \times \vec{F}_1$$

$$\vec{\tau}_2 = \vec{r}_2 \times \vec{F}_2 = -r_2 \times F_1$$

$$\tau = \vec{r}_1 \times \vec{F}_1 - \vec{r}_2 \times \vec{F}_1$$

$$= (\vec{r}_1 - \vec{r}_2) \vec{F}_1 \qquad (32)$$

Summary

In the table below, $x_0$ and $\theta_0$ are chosen to be zero for easy parametrizing. However, for the angular quantities, two possible directions of rotation ~~about~~ the fixed axis are chosen arbitrarily to be the direction in which $\theta \uparrow$ ie

$$\omega = \frac{d\theta}{dt} \; ; \quad \text{if } \theta \uparrow \text{ as } t \uparrow, \; \omega \text{ is +ve}$$

$$a = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2} \; ; \quad \text{if } \omega \uparrow \text{ as } t \uparrow, \; a \text{ is +ve.}$$

| TRANSLATIONAL MOTION (Fixed direction) | ROTATIONAL MOTION (Fixed Axis) |
| --- | --- |
| $v = v_0 + at$ | $\omega = \omega_0 + at$ |
| $x = v_0 t + \frac{1}{2}at^2$ | $\theta = \omega_0 t + \frac{1}{2}at^2$ |
| $v^2 = v_0^2 + 2ax$ | $\omega^2 = \omega_0^2 + 2a\theta$ |

ASSIGNMENT

1. Calculate the angular velocity of a disc rotating with uniform motion of 13.2 rad every 6s. Calculate the period and frequency of rotation.
   How long will it take the disc to rotate through an angle of 780° and make 12 revolutions
2. Find the magnitude of the velocity and the centripetal acceleration of the earth in its motion round the sun. Radius of the earth's orbit is $R = 1.49 \times 10^{11}$ m and its period of revolution is $3.16 \times 10^7$ s.

# Newton's Law of Gravitation

DEFN: The gravitational interaction between two bodies can be expressed by an attractive central force proportional to the masses of the bodies and inversely proportional to the square of the distance between them.

## Relationship B/W g and the mass of the Earth

$$F = \frac{GMm}{R^2} \quad (2)$$

M := mass of the earth
m := " " a particle on earth's surface
R := distance of the particle from earth's centre
≡ earth's radius.

We recall that

$$F = mg$$

$$\therefore mg = \frac{GMm}{R^2}$$

$$g = \frac{GM}{R^2} \quad \cdots (3)$$

Remark

(1) m does not appear in the equation implying that all bodies (irrespective of their masses) should fall with ~~the same~~ accl$^n$ in agreement with expt'l obsern (neglecting air resistance) ie

$$M = \frac{g}{G} R^2 \quad \cdots (3a)$$

(2) Implication. The force on mass m is implicitly assumed to be the same as if all the mass of the Earth is concentrated at its centre due to the fact that m is assumed to be at a distance R from the centre of the earth.

## GRAVITATIONAL INTERACTION

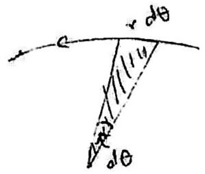
### INTRODUCTION

Originally the earth was thought to be at the centre of the universe relative to which all other planets revolve. This idea was however modified by Copernicus who put the Sun at the geometrical centre of the universe. This aids Kepler to discover the laws of planetary motion. ~~The laws~~

### KEPLER'S LAWS OF PLANETARY MOTION.

1. The orbit of each planet is an ellipse with the sun at one focus.

2. The position vectors of any planet relative to the sun sweeps out equal areas of its ellipse in equal times. [ This is often referred to as law of areas ie $\frac{1}{2}r^2\dot{\theta}$ = constant ]



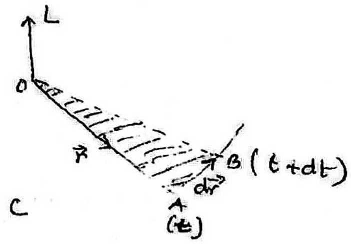
Area swept in time $dt = \frac{1}{2}r^2 d\theta$

Area velocity $= \frac{1}{2}r^2\frac{d\theta}{dt} = \frac{1}{2}r^2\dot{\theta}$

$$\epsilon = \sqrt{1 + \frac{2El^2}{mk^2}}$$

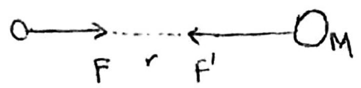
| | | |
|---|---|---|
| $\epsilon < 1$ | $E < 0$ | → ellipse |
| $\epsilon = 1$ | $E = 0$ | → parabola |
| $\epsilon > 1$ | $E > 1$ | - hyperbola |
| $\epsilon = 0$ | $E = -\frac{mk^2}{2l^2}$ | - circle. |

ALTERNATIVELY



## LAW OF GRAVITATION

This law simply considers the interactions between two bodies (be it planets or small particles) which produces a motion that can be described by Kepler's laws.




### FEATURES

1. The force acts along the line joining the two interacting bodies and if it assumed that gravitational interaction is universal property of all matter, then the force F associated with the interaction must be proportional to the amount of matter in each body ie their respective masses. We may thus write

$$F = mM\,f(r)$$

2. It is extremely hard to predict the kind of relationship between the force F and the distance between the two masses due to the fact that gravitational interaction is extremely weak thus requiring very sensitive experimental set up

3. F is very small except in cases where the masses are very large (such as two planets). On the other hand, distances between them should be very small. However, this has a major snag that at such short distances other stronger interactions come into play masking the gravitational effect.

4. One can only infer that gravitational interaction is attractive and varies inversely with the square of the distance between the two bodies ie

$$f(r) \propto \frac{1}{r^2} \quad \text{or} \quad f(r) = G/r^2$$

$$F = G\frac{Mm}{r^2} \qquad (1)$$

r := distance b/w m & M

G := universal gravitational constant $= 6.67 \times 10^{-11}\ Nm^2kg^{-2}$

## CENTRIPETAL ACCELERATION

Centripetal acceleration, a is

$$a = \frac{v}{t} \quad \left[\frac{m}{s \cdot s} = \frac{m}{s^2}\right] \qquad (24)$$

using (23), we have

$$a = \frac{\omega r}{t}$$

and using (23) once again to eliminate ω, we have

$$a = \frac{v \cdot v}{r} = \frac{v^2}{r} \qquad (25)$$

or

$$a = \omega^2 r \qquad (26)$$

Centripetal acceleration is always directed towards the centre of the circle.

## CENTRIPETAL FORCE

The force required to keep an object moving in a circle of radius r called the centripetal force is

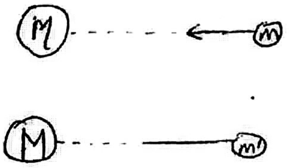
$$F = ma = \frac{mv^2}{r} = m\omega^2 r \qquad (27)$$

The force is directed towards the centre of the cir

contradicts experimental result that $g$ is the same for all bodies. Thus $mg$ and $m$ can be used indiscriminately since both are indistinguishable.

In comparing masses of two bodies, we need a third body as reference. The reference mass $M$ is usually taken as the mass of the earth.

eg



$$F = \frac{GMm}{R^2}$$

$$F' = \frac{GMm'}{R^2}$$

$$\therefore \boxed{\frac{F}{F'} = \frac{m}{m'}} \qquad (7)$$

Remark

Equation (7) provides a good method for comparing forces without necessarily measuring each one. The principle of the balance allows us to use this method when the reference body is the earth. The balance achieves equilib when the two forces are equal and ~~therefore the~~ masses are equal

# Mass - Energy Equivalence

In physics, mass - energy equivalence is the principle that anything having mass has an equivalent amount of energy & vice versa. This quantities are directly related to one another by Einstein's formula;

$$E = mc^2 \quad \text{—— (15)}$$

## Remark

1. Because speed of light is a large number in everyday units, the formula implies that even an everyday object at rest with modest amount of mass has a very large amount intrinsically.

2. A consequence of the mass - energy equivalence is that if a body is stationary, it still has some internal or intrinsic energy called the "Rest Energy" corresponding to the "Rest Mass". This rest mass is also called the "Intrinsic ~~or Equivalent~~ Mass" because it remains the same regardless of this motion.

3]. When the body is in motion its total energy is greater than its rest energy and equivalently its total mass is greater than its rest mass. The total mass in this contest is called the relativistic mass.

4] The mass - energy formula also serves to convert units of mass to units of energy (and vice-versa), no matter what system of measurement units is used.

$$E = mc^2 \left[ kg \cdot \frac{m^2}{s^2} = 1J \right]$$

which is the unit of Energy

$$m = \frac{E}{c^2} \left[ kg\frac{m^2}{s^2} / m^2/s^2 = \frac{kg\,m^2 s^2}{m^2 s^2} = 1\,kg \text{ - which is the unit of mass} \right]$$

## Power

Power, P is defined as

$$P = \frac{\text{Workdone}}{\text{Time taken}} = \frac{E}{t} \left[\frac{J}{S} = [W]\right] \quad (11)$$

## MECHANICAL EFFICIENCY

Mechanical Efficiency is the measure of the effectiveness with which a mechanical system performs.

$$\eta = \frac{\text{Power delivered by a mechanical System}}{\text{Power applied to it}} \quad (12)$$

**Remark;**

1. Due to friction $\eta \leq 1$
2. For simple machines such as lever and the jack screw, the efficiency $\eta$ is

$$\lambda_s = \frac{\text{Actual load lifted}}{\text{Theoretical force applied}}$$

## Conservative Forces

The workdone by an object is independent of the path taken. This simply means that the total workdone in a closed path is zero i.e work expended on one part of the closed path is regained in the other or remaining part.

Whenever the work done in moving round a closed path in a field to the original point is zero then the forces in the field are called conservative force of earth's gravitational field.

## Non-Conservative Forces

If the work done in moving round a closed path to the original field is not zero, the force is said to be non-conservative of friction, air resistance.

## Principle of Conservation of Energy

This states that the total energy in a closed system is always Conserved.

# WORK, ENERGY AND POWER

Work is defined as;

$$W = F \times s \quad \text{— (5)}$$

where $W$ - workdone

$F$ - applied force

$s$ - distance travelled in the direction of the force.

However, if the force acts on an object at an angle to the distance travelled, then

$$W = F\cos\theta \times s \quad \text{— (6)}$$

## Energy

It can be classified into two viz

### Kinetic energy

This is the energy possessed by virtue of motion. It formula can be derived from the third equation of motion. e.g consider a body moving with velocity $V_0$ and is gradually brought to rest in a distance by a constant force acting against it. In this case $V = 0$ and from

$$V^2 = V_0^2 - 2as$$

$\Rightarrow 0 = V_0^2 - 2as$ - The negation sign is because the motion decelerates

$$V_0^2 = 2as$$

$$\text{or } as = \frac{V_0^2}{2} \quad \text{— (7)}$$

Using equation (5)

$$W = Fs$$

but $F = ma$

$$W = Mas$$

and using equation (7) we have

$$\boxed{W = \frac{MV_0^2}{2} = T} \quad \text{— (8)}$$

where $T$ - kinetic energy.

Unit - Joule [J]

$$1J = 1\,\frac{kg\,m^2}{s^2}$$

### Potential energy

It is the energy possessed by virtue of position.

from (5)

$$W = Fs$$

but $s = h$ = height

$$W = Fh$$

$F = ma = mg$ [because $a$ = acceleration due to gravity]

$\boxed{W = mgh = V}$ — (9) where $V$ is the potential energy

Total Energy, $E = T + V$ — (10)

(c) To check the correctness of the equations above, Newton compared the centripetal acceleration of the moon with $g$ ie

centripetal acceleration of the moon $a$ is

$$a_c = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2}$$

If $r = 3.84 \times 10^8 m$, $T = 2.36 \times 10^6 s$

then $a_c = 2.72 \times 10^{-3} ms^{-2}$

where $T$ denotes the period of rotation of the moon in which case the moon month = 27.3 days.

$$\frac{g}{a_c} = 3602 \approx (60)^2$$

but $R$ = radius of the earth = $6.37 \times 10^6 m$

$$\left(\frac{r}{R}\right)^2 = \left(\frac{384}{6.34}\right)^2 \approx (60)^2$$

$$\Rightarrow \frac{g}{a_c} = \left(\frac{r}{R}\right)^2 \quad (6)$$

and within the limits of this calculation the two accelerations are in inverse proportion to the square of the distances of the points from the centre of the earth.

Proof

$$f_c = m'g' = m'w^2 r$$

$F_c$ := centripetal force of the moon

$$m'g' = \frac{G m' M}{r^2}$$

PHY 101 Linear Momentum

Linear momentum, P can be written as

$$P = MV \quad \text{— (1)}$$

For an object having a mass m and if;

u = initial velocity of the object

v = final velocity of the object

then the change of momentum = $MV - MU = m(v-u)$ — (2)

The force, F acting on the object for a time, t is given by

$$F = \frac{m(v-u)}{t} = \frac{\text{Change in momentum}}{\text{time}}$$

$Ft = m(v-u)$; Change in momentum — (3)

$Ft$ = Impulse of the force on the object.

Principle of Conservation of Linear Momentum

The principle states that in the absence of external force acting on a system of colliding bodies, the total momentum of the objects remains constant.

$$M_1U_1 + M_2U_2 = M_1V_1 + M_2V_2$$

Or

$$M_1U_1 + M_2U_2 - (M_1V_1 + M_2V_2) = 0 \quad \text{— (4)}$$

Where;

$M_1$ & $M_2$ are the masses of A and B respectively

$U_1$ & $U_2$ are the initial velocities of their bodies before collision.

$V_1$ & $V_2$ are the final velocities of the bodies after collision..

REMARK;

The impulse is the same for both bodies and acts at the time in opposite direction i.e this is Newton's third law which states that forces occur in pairs: The pair of forces are equal in magnitude but opposite in direction.

$$g' = \frac{GM}{r^2}$$

For bodies falling near the centre of the earth,

$$g = \frac{GM}{R^2} \quad := \text{ density of the } ~~\text{great}~~ \text{ gravitational field}$$

$R :=$ radius of the earth.

(*) Total flux at some point distance $R$ from $M$ is ( $\rho \times V = M$ ) ie

$$g\,4\pi r^2 = \frac{GM}{R^2} \times 4\pi R^2$$

$$= 4\pi GM$$

$$\frac{g}{g'} = \frac{\frac{GM}{R^2}}{\frac{GM}{r^2}} = \left(\frac{r}{R}\right)^2$$

$$\therefore \frac{g}{a_c} = \left(\frac{r}{R}\right)^2$$

where $g' = a_c$.

JUSTIFICATION OF THE EQUAL ARM BALANCE TECHNIQUE (USE OF CHEMICAL BALANCE)

Eqn (8.2) is independent of mass of the falling object or stated in another way, the inertial mass of a falling object and the gravitational mass are the same. This statement is implicitly assumed in the constancy of G of assuming $m_g :=$ gravitational mass and $m :=$ inertial mass then

$$mg = \frac{G m_g M}{R^2}$$

$$g = \left(\frac{m_g}{m}\right) \frac{GM}{R^2}$$

$\Rightarrow$ $\left(m_g/m\right)$ is not the same for all bodies hence $g$ would be different for each body and this and substituting (21) in (22), we have

$$\omega = \frac{v}{r}$$

or $v = \omega r$

$$\vec{V} = \vec{\omega} \times \vec{r}$$

=> $\vec{V}$ is perpendicular to the distance formed by $\vec{\omega}$ and $\vec{r}$

## Centripetal Acceleration

Centripetal acceleration, $a$ is

$$a = \frac{v}{t} \left[\frac{m}{s \cdot s} = \frac{m}{s^2}\right] \quad \text{— (23)}$$

Using 23, we have

$$a = \frac{\omega r}{t} \quad \text{— (24)}$$

and using (23) once again to eleminate $\omega$, we have

$$a = \frac{v\,v}{r} = \frac{v^2}{r} \quad \text{— (25)}$$

or

$$a = \omega^2 r \quad \text{— (26)}$$

entripetal accelation is always directed towards the centre of the
rcle.

## entripetal Force

## Circular Motion

When an object moves in a circle at a constant speed, its velocity (which is a vector) is constantly changing. Its velocity is changing not because the magnitude of the velocity is changing but because the direction is.

This constantly changing velocity means that the object is accelerating [Centripetal acceleration]. For this acceleration to happen, there must be a resultant force which is known as Centripetal force.

## Angular speed

$$w = \frac{\alpha}{t} \quad — (14)$$

$w$ = angular speed of the object
$\alpha$ = angle it moves through measured in degree.
$t$ = time.

When the circle is completed i.e when $\alpha = 360^\circ$

$$\alpha = 2\pi \quad — (15) \quad \pi = 180^\circ$$

$$\therefore w = \frac{2\pi}{T} \quad — (16)$$

$$\Rightarrow T = \frac{2\pi}{w} \quad — (17)$$

$$\text{but } T = \frac{1}{F} \quad — (18)$$

$$F = \frac{w}{2\pi} \quad — (19)$$

where T – period
F – resultant frequency.

When $\alpha$ is expressed in radians,

$$\alpha = \frac{s}{r} \quad — (20)$$

$$\text{but } v = \frac{s}{t} \quad — (21)$$

by using equation (14), we have

$$w = \frac{s}{rt} \quad — (22)$$

Hydrodynamics


The figure above - the pipe

- The pipe is allowed to have a changing ~~cross-sectional~~ cross-sectional area and a changing height above some reference level.
- By doing a certain net amount of work W on ~~the~~ fluid, there is a change in ~~the~~ K.E & P.E of the fluid i.e.

$$W = \Delta T + \Delta E_p$$

- Work is done on the fluid by pushing with a force $F_1$ on a quantity of the fluid at position 1 and moving each small element of the volume of fluid through a net displacement $s_1$ at a constant velocity $v_1$. Thus a volume

$$V_1 = A_1 s_1$$

has been displaced a distance $s_1$ and

$$W = F_1 s_1$$

where $F_1$ is force

The question is what is the effect of this displacement on the fluid at position 2? Because the fluid is incompressible, the displacement of a volume $A_1 s_1$ through a distance $s_1$ at position 1 means that there must be an equal volume displacement at position 2. Therefore, a quantity of fluid with a cross-sectional area $A_2$ and a length $s_2$ is displaced a distance $s_2$ at a constant velocity $v_2$ where

$$A_1 s_1 = A_2 s_2$$

The volume of fluid at position 2 moves along the pipe only by pushing on the next element of fluid. This exerts $F_2$ acts through a distance $s_2$, so an amount of work $F_2 s_2$ is done by this volume of fluid.

∴ The net work done, $W = F_1 s_1 - F_2 s_2$

$$= P_1 A_1 s_1 - P_2 A_2 s_2$$

$$= (P_1 - P_2) V$$

where

$F = P \times A$ (i.e. $P$ = pressure, $A$ = area)

and $V = V_1 = A_1 s_1 = V_2 = A_2 s_2$

(5)

The net effect of the changes caused by pushing on the fluid with the force $F_1$ is exactly the same as the transfer of a mass $\rho V$ of fluid from position 1 to position 2 i.e. a mass $\rho V$ (where $\rho$ is the density of the fluid) of fluid at a pressure $P_1$ moving with a velocity $v_1$ at a height $h_1$ is transformed into an equal mass of fluid at a pressure $P_2$, moving with a velocity $v_2$ at a height $h_2$. Then, the changes in the potential and kinetic energies between position 1 and 2 are

$$\Delta E_p = \Delta mgh = mg\Delta h = \rho V g (h_2 - h_1)$$

$$\Delta T = \Delta\left(\tfrac{1}{2}mv^2\right) = \tfrac{1}{2} m \Delta v^2 = \tfrac{1}{2}\rho V (v_2^2 - v_1^2)$$

Since

$$W = \Delta T + \Delta E_p$$

$$(P_1 - P_2) V = \rho V g (h_2 - h_1) + \tfrac{1}{2}\rho V (v_2^2 - v_1^2)$$

Dividing by V and rearranging we have

$$P_1 + \rho g h_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \tfrac{1}{2}\rho v_2^2$$

In general we have

$$\boxed{P + \frac{\rho v^2}{2} + \rho g h = \text{constant}} \quad (2)$$

* It is inferred from Bernoulli's law/eqn is the sum of the pressure at any point and the kinetic energy per unit volume and the potential energy per unit volume are constant.

APPLICATIONS

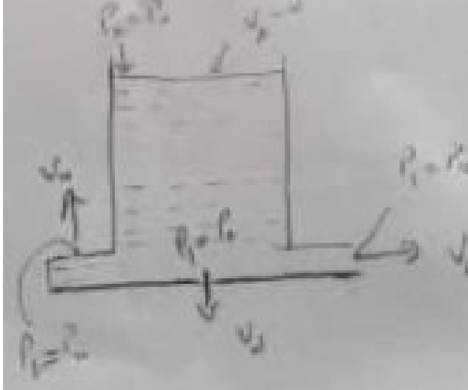
HYDROSTATICS

For static fluids Bernoulli's eqn becomes

$$(P_1 - P_2) = \rho g (h_2 - h_1)$$

because the velocity term disappears.

TORRICELLI'S LAW




* The figure above
  The aim is to determine the three velocities $v_a, v_b, v_c$ as it leaves these three holes.
* The upper surface of the liquid corresponds to position 2 in Bernoulli's eqn where the atmospheric pressure $P_2 = P_a$
* Since the liquid is flowing out of the tank, it follows that the upper surface will be falling
* If the hole is made small enough, then the

(C)

the rate of flow of the liquid is very small so that we can write

$$v_2 = 0$$

* Position 1 corresponds to the bottom of the tank. Just outside the hole where the flow velocity are measured, the pressure is also atmospheric i.e

$$P_1 = P_0$$

$$P_0 + \rho g h_1 + \tfrac{1}{2}\rho v_1^2 = P_0 + \rho g h_2$$

so that

$$v_1 = \sqrt{2g(h_2 - h_1)}$$

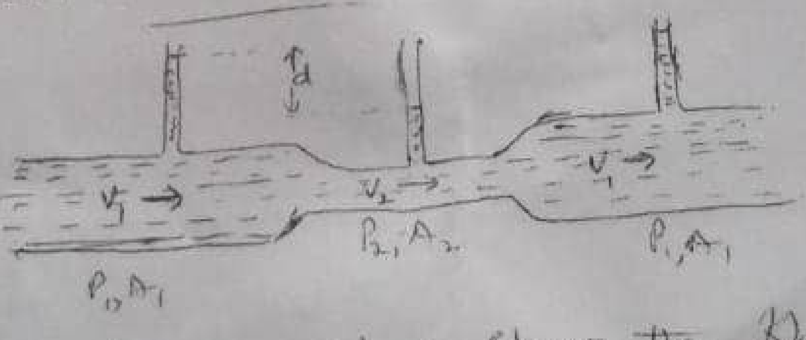
which is Torricelli's eqn

Remark

$v_1$ does not depend on the direction of flow i.e

$$v_1 = v_a = v_d = v_n$$

FLOW AT CONSTANT HEIGHT.




The figure above shows the flow of a liquid through a horizontal pipe that has a varying

(7)

cross-sectional area. With $h_1 = h_2$ Bernoulli's eqn becomes

$$P_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \tfrac{1}{2}\rho v_2^2$$

Because the pipe is constricted at the middle we have $A_2 < A_1$ and $v_2 > v_1$. It then follows that we can infer from Bernoulli's eqn above that $P_2 < P_1$

The difference $d$ ~~in the heights~~ to which the liquid rises in the side tubes serves to measure the pressure difference.

$$P_1 - P_2 = \rho g d.$$