UNIVERSITY OF ABUJA
PHYSICS DEPARTMENT
PHY 204 TEST

Q1(a) Two concentric metal spheres of radii x and y (x > y) have the larger sphere earthed. Drive an expression for the capacitance of the system.

(b) Three concentric metal spheres have radii 10cm, 20cm, 30cm. The outer sphere is earthed whilst the inner one is charged to a potential of 1000V, the intermediate sphere being isolated and uncharged. Find the charge on this inner sphere.

PHY 204 Test Solution

$(4\pi\varepsilon_0)^{-1} = 9 \times 10^{-9}\ Nm^2/C^2$

(a) $V = V_+ + V_-$

$= \frac{q}{4\pi\varepsilon_0 y} - \frac{q}{4\pi\varepsilon_0 x}$

$4\pi\varepsilon_0 = \frac{1}{9\times10^{-9}}$

$= \frac{q}{4\pi\varepsilon_0}\left\{\frac{1}{y} - \frac{1}{x}\right\}$

$= \frac{q}{4\pi\varepsilon_0}\left\{\frac{x-y}{yx}\right\}$

$C = \frac{q}{V} = \frac{4\pi\varepsilon_0 yx}{x-y}$

(b) $C = \frac{4\pi\varepsilon_0 (0.1 \times 0.3)}{(0.3 - 0.1)}$

$= \frac{0.03\, 4\pi\varepsilon_0}{0.2}$

$\therefore \frac{q}{V} = \frac{0.03\, 4\pi\varepsilon_0}{0.2}$

$q = \frac{0.03\, 4\pi\varepsilon_0}{0.2} \times V$

$= 1.668 \times 10^{-8}\ C$

PHY 204

SOLUTION TO ASSIGNMENT 6

I. At node V_1

$$7 - 3V_1 - 5 - 1(V_1 - V_2) = 0$$

$$4V_1 - V_2 = 2 \qquad (1)$$

At node V_2

$$5 + (V_1 - V_2) - 3V_2 - 2(V_2 - V_3) = 0$$

$$-V_1 + 6V_2 - 2V_3 = 5 \qquad (2)$$

At node V_3

$$2(V_2 - V_3) + 17 - V_3 - 4V_3 = 0$$

$$-2V_2 + 7V_3 = 17 \qquad (3)$$

These equations may be solved for the node voltages using one of the variety of methods for simultaneous equations i.e from eqn (1)

$$V_1 = \frac{V_2 + 2}{4} \qquad (4)$$

Substituting for V_1 in (2) we have

$$-\left(\frac{V_2 + 2}{4}\right) + 6V_2 - 2V_3 = 5$$

$$-\frac{V_2}{4} - \frac{1}{2} + 6V_2 - 2V_3 = 5$$

$$\frac{23}{4}V_2 - 2V_3 = 5\tfrac{1}{2} \qquad (5)$$

Solving (3) and (5) simultaneously, we have

$$-2V_2 + 7V_3 = 17 \qquad \times 2$$

$$\tfrac{23}{4}V_2 - 2V_3 = \tfrac{11}{2} \qquad \times 7$$

Scanned by CamScanner

$$-4V_2 + 14V_3 = 34 \quad (6)$$

$$\frac{161}{4} V_2 - 14V_3 = 77\tfrac{1}{2} \quad (7)$$

(6) + (7) gives

$$36\tfrac{1}{4} V_2 + 72\tfrac{1}{2}$$

$$V_2 = 2V$$

Substituting (8) into (4) we have

$$V_1 = \frac{V_2 + 2}{4} = 1V.$$

V_3 also yields from (3)

$$-4 + 7V_3 = 17$$

$$V_3 = 3V.$$

ASSIGNMENT II

At Mesh I

$$-V_{g_1} + R_1 i_1 + R_2(i_1 - i_2) + R_3(i_1 - i_3) = 0$$

$$(R_1 + R_2 + R_3) i_1 - R_2 i_2 - R_3 i_3 = V_{g_1}$$

At mesh II

$$i_2 R_4 + V_{g_2} + R_5(i_2 - i_3) + R_2(i_2 - i_1) = 0$$

$$(R_2 + R_4 + R_5) i_2 - R_2 i_1 - R_5 i_3 = -V_{g_2}$$

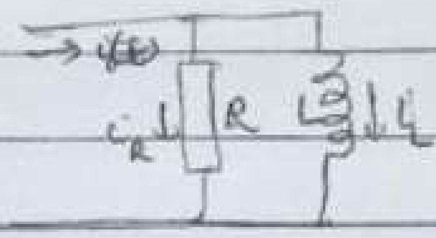
At mesh III

$$i_3 R_6 - V_{g_1} + R_3(i_3 - i_1) + R_5(i_3 - i_2) = 0$$

$$(R_3 + R_5 + R_6) i_3 - R_3 i_1 - R_5 i_2 = V_{g_1}$$

Assignment VI

RL parallel



$v = V_m \cos \omega t$

$i_T = i_R + i_L$

$= \frac{v}{R} + \frac{1}{L}\int v\,dt$

$= \frac{V_m \cos \omega t}{R} + \frac{V_m \sin \omega t}{\omega L}$

$\therefore i_T = A \cos(\omega t - \phi)$

where

$$A = V_m \sqrt{\frac{1}{R^2} + \left(\frac{1}{\omega L}\right)^2}$$

$$\phi = \tan^{-1} \frac{R}{\omega L}$$

R-C parallel

$v = V_m \sin \omega t$

$i_T = i_R + i_C$

$= \frac{v}{R} + C\frac{dv}{dt}$

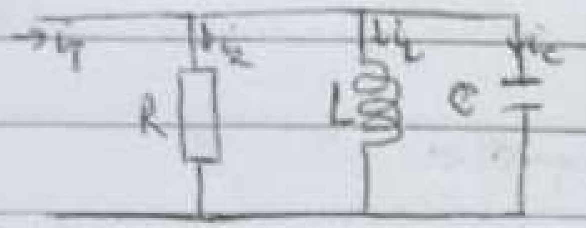
$= \frac{V_m \sin \omega t}{R} + \omega C V_m \cos \omega t$

$= A \sin(\omega t + \phi)$

$$A = \sqrt{\frac{1}{R^2} + (\omega C)^2}\; V_m$$

$$\phi = \tan^{-1} \omega C R$$

R-C-L PARALLEL



$= v = V_m \sin \omega t$

$$i_T = i_R + i_L + i_C.$$

$$= \frac{v}{R} + \frac{1}{L}\int v\,dt + C\frac{dv}{dt}$$

$$i_T = \frac{V_m \sin \omega t}{R} - \frac{V_m \cos \omega t}{\omega L} + \omega C V_m \cos \omega t$$

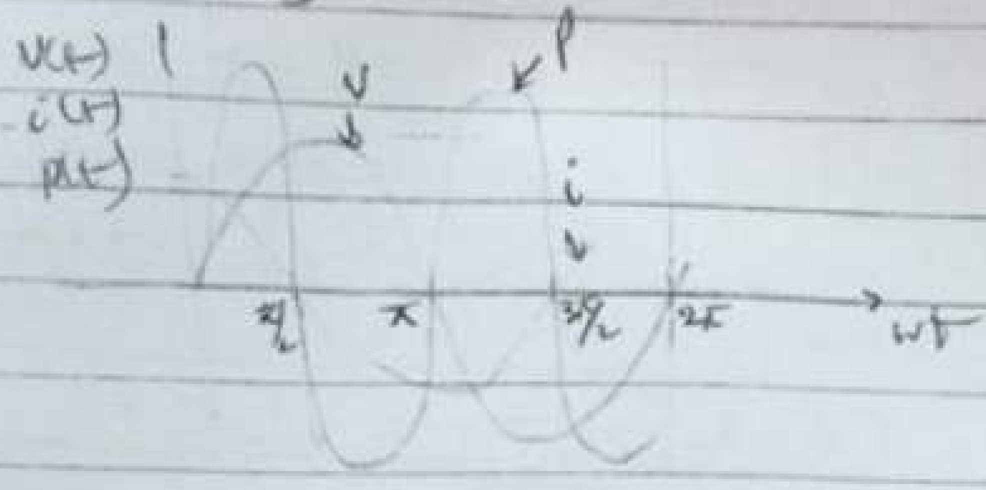
$$i_T = A \sin(\omega t + \phi)$$

where

$$A = \sqrt{\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2}\, V_m$$

$$\phi = \tan^{-1} \frac{\left(\omega C - \frac{1}{\omega L}\right)}{R}.$$

ASSIGNMENT IV



PURE CAPACITOR

Here i leads v by $\pi/2$

PURE RESISTOR

Consider a pure resistor with

$v = V_m \sin \omega t$

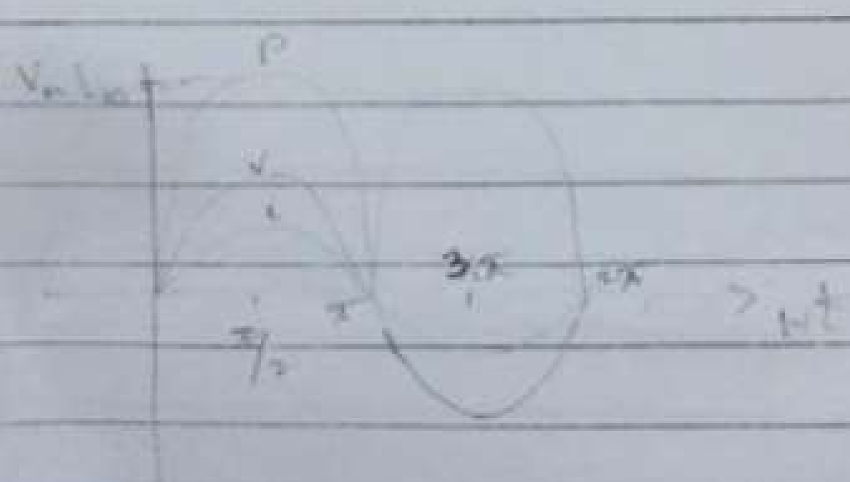
$i = I_m \sin \omega t$

Here v & i are in phase

$p = vi = V_m I_m \sin^2 \omega t$

$\sin^2 \omega t = \frac{1}{2}(1 - \cos 2\omega t)$

$\therefore P = \frac{1}{2} V_m I_m (1 - \cos 2\omega t)$.



P has twice the frequency of v or i

P is always +ve varying from zero to a maximum value of $V_m I_m$

The average value of p over the entire cycle is $\frac{1}{2} V_m I_m$

Generally when $V = V_m \sin \omega t$ is applied to a network, the resulting current $i = I_m \sin(\omega t + \theta)$

The phase angle θ can be positive (i.e. Capacitor) or negative (inductive).

$$P = V_m I_m \sin \omega t \sin(\omega t + \theta)$$

$$\sin A \sin B = \frac{1}{2}\left\{\cos(A-B) - \cos(A+B)\right\}$$

$$\cos -A = \cos A$$

$$\therefore P = \frac{1}{2} V_m I_m \left\{\cos\theta - \cos(2\omega t + \theta)\right\}$$

P consists of a sinusoidal term $-\frac{1}{2} V_m I_m \cos(2\omega t + \theta)$ which have an average power value of zero and a constant term $\frac{1}{2} V_m I_m \cos\theta$