UNIVERSITY OF ABUJA
FACULTY OF SCIENCE
DEPARTMENT OF MATHEMATICS
2016/2017 SECOND SEMESTER EXAMINATION
MTH 102 - DIFFERENTIAL CALCULUS AND ITS APPLICATIONS UNIT: 3 TIME: 3 HOURS
INSTRUCTION: ANSWER ALL QUESTIONS IN SECTION A AND ANY TWO QUESTIONS IN SECTION B

PAST QUESTION & SOLUTIONS

1. Let the function $f(x)$ be defined as $f(x) = x^2 + 13x - 12$ and if $x = 3$ find its limit
(a) 56 (b) 36 (c) 32 (d) 10
2. Let the function $f(x)$ be defined as $f(x) = (-x)^2 + 2x - 21$ and if $x = 2$ find its limit
(a) 56 (b) 36 (c) 32 (d) 10
3. $f(x) = \cos x$ is
(a) EVEN (b) ODD (c) PERIODIC (d) NEITHER ODD NOR EVEN
4. $f(x) = x^3 + 1$ is
(a) EVEN (b) ODD (c) PERIODIC (d) NEITHER ODD NOR EVEN
5. $f(x) = \tan x$
(a) EVEN (b) ODD (c) PERIODIC (d) NEITHER ODD NOR EVEN
6. Evaluate the limit of $f(t) = 3t^3 + 2t$ at the point t=2 (a) 37 (b) 39 (c) 28 (d)38
7. The derivative of $f(x) = \ln x$ is
(a) $1/x^2$ (b) $1/x$ (c) x^2 (d) x^3
8. The derivative of $\frac{d(e^{3x^2})}{dx}$ is (a) $1/x^2 e^{3x^2}$ (b) $1/x e^{3x^2}$ (c) $6xe^{3x^2}$ (d) $3x^2 e^{3x^2}$
9. Evaluate $\lim_{x \to x_0} \frac{x^2 - x_0^2}{x - x_0}$ (a) $x^2 - x_0^2$ (b) $x - x_0$ (c) $2x_0$ (d) $2(x - x_0)$
10. Evaluate $\lim_{x \to 1} \frac{x^n - 1}{x - 1}$ (a) x_n (b) 0 (c) $2x_0$ (d) 2
11. The integral $\int \tan x dx$ is
(a) $\ln|\sec x| + c$ (b) $\tan(x - x_0) + c$ (c) $\cot x + c$ (d) $\sec x + c$.
12. The integral $\int \sec x \tan x dx$ has a solution (a) $\sec x + c$ (b) $\tan x + c$ (c) $\cos x + c$ (d) $\sec^2 x + c$
13. $\int \sec^2 x dx$ is (a) $-\sin xc$ (a) $\tan x + c$ (b) $\tan^2 x + c$ (c) $\cos x + c$ (d) $\sec^2 x + c$
14. $\int \sin x dx$ is (a) $-\sin xc$ (b) $\tan^2 x + c$ (c) $\cos x + c$ (d) $-\cos x + c$
15. The gradient of a curve is $6x + 2$ and it passes through the point (1, 3). Find its equation.
(A) $y = 3x^2 - 2$ (B) $y = 2x - 5$ (C) $y = 3x^2 + 2x - 2$ (D) $y = x^2 + 2x - 2$
16. Find $\frac{dy}{dx}$ if $y = x^{2x}$ (A) $x(\ln x + 1)$ (B) $2x(\ln x - 1)$ (C) $2x^{2x}(\ln x + 1)$ (D) $x^{2x} \ln x$
Use the point- slope formula $y - y_0 = m(x - x_0)$ for representing the equation of a line to answer questions 17-18
17. Find the slope of a line passing through $(-2,12)$ (a)4 (b)6 (c)16 (d) 8
18. Find the tangent line through the point $(3,7)$ (A) 6 (B)-6 (C)8 (D)
19. Evaluate $\int f(x)f'(x)dx$ (A) $\frac{[f(x)]^2}{2} + k$ (B) $f'(x) + k$ (C) $\frac{\ln(f(x))}{2} + k$ (D) $f(x)f'(x) + k$
20. Evaluate $\int \frac{\sin x}{1+\cos x} dx$ (A) $-\ln(1 + \cos x) + k$ (B) $\tan x + k$ (C) $\ln(1 + \cos x) + k$ (D) $\ln(\sin x) + k$
21. We evaluate $\int \frac{dx}{\sqrt{a^2-x^2}}$ using the substitution (A) $x = a\sec\theta$ (B) $x = a\tan\theta$ (C) $x = a\sin\theta$ (D) $x = a\cos\theta$
22. Find $\frac{dy}{dx}$ for $y = 3^x$ (A) $3^x \ln 3$ (B) $\frac{3^x}{\ln 3}$ (C) $x3^{x-1}$ (D) 3^{x^2}
23. Evaluate $\lim_{x \to -1} x^2 - 5x + 2$ (A) -5 (B) 8 (C) 2 (D) -8
24. Find $\frac{dy}{dx}$ if $x = 1 + t^2$ and $y = \frac{1}{t}$ at point $t = 2$. (A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) $-\frac{1}{16}$ (D) -4

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25. Find $\frac{dy}{dx}$ if $y = \frac{\sqrt{3x+1}}{4x}$
(A) $\frac{(3x-4)\sqrt{3x+1}}{16x^2(3x+1)}$ (B) $\frac{3(3x+1)}{16x^2}$ (C) $\frac{(4-3x)\sqrt{3x+1}}{16x^2}$ (D) $\frac{(3x+4)(x-1)}{16x^2}$

26. Let $y = f(x)$ then $y + \Delta y$ is (A) $f(x) + \Delta y$ (B) $f(x + \Delta x)$ (C) $f(x) + \Delta y$ (D) $f(x + \Delta y)$

27. Obtain y'' for $y = 2x^3 - 3x^2 + 1$ (A) $6x^2 - 6x$ (B) $12x$-6 (C) 12 (D) $6x^2 + 1$If $x^2 - xy = 1$ find $\frac{dy}{dx}$ (A) $\frac{y-2x}{-x}$ (B) $\frac{2xy}{3}$ (C) $\frac{2x-y}{-x}$ (D) $2x - xy$

28. Obtain $f'(x)$ for $f(x) = e^{2x+5}$ (A) $2e^{2x+5}$ (B) $\frac{e^{2x+5}}{2}$ (C) e^{2x+5} (D) e^{2}

29. Find $\frac{dy}{dx}$ if $y = \frac{\sqrt{3x+1}}{4x}$
$\frac{(3x-4)\sqrt{3x+1}}{16x^2(3x+1)}$ (B) $\frac{3(3x+1)}{16x^2}$ (C) $\frac{(4-3x)\sqrt{3x+1}}{16x^2}$ (D) $\frac{(3x+4)(x-1)}{16x^2}$

30. The derivative of $y = \frac{f(x)}{g(x)}$ is (A) $\frac{g(x)f'(x)-f(x)g'(x)}{(g(x))^2}$ (B) $\frac{f(x)g(x)}{g'}$ (C) $\frac{f'(x)}{g'(x)}$ (D) $\frac{f(x)g'(x)-f'(x)g(x)}{(g(x))^2}$

31. Evaluate $\frac{d}{dx}(\ln(3x + 2))$ (A) $\frac{3}{3x+2}$ (B) $\frac{1}{3x+2}$ (C) $3x + 2$ (D) $\frac{1}{3}$

32. Evaluate $\int coshx dx$ (A) $-sinhx + k$ (B) $sinhx + k$ (C) $\frac{cosh^2x}{2} + k$ (D) $\frac{sinh^2x}{2} + k$

33. Find $\frac{dy}{dx}$ when $x = t^2$ and $y = t^3$ (A) $\frac{3}{2}t$ (B) $3t^2$ (C) $2t$ (D) $\frac{2}{3}t$

34. Which of the following is incorrect (A) $sin^2x = \frac{1-cos2x}{2}$ (B) $cos^2x + sin^2x = 1$ (C) $sin2x = 2sinxcosx$ (D) $cosh^2 + sinh^2x = 1$

35. Differentiate $= sinh\frac{x}{2}$. (A) $\frac{1}{2}cosh\frac{x}{2}$ (B) $cosh\frac{x}{2}$ (C) $sinh^2\frac{x}{2}$ (D) $-\frac{1}{2}cosh\frac{x}{2}$

36. The displacement of a body travelling in a straight line is given by $s = t^2 - 5t + 6$, when is the body momentarily at rest? (A) $t = \frac{5}{2}sec$ (B) $t = \frac{2}{5}sec$ (C) $t = 0sec$ (D) $t = 5\ sec$.

37. Find the area enclosed by the curves $y = x^2$ and $y^2 = 8x$ (A) $\frac{7}{3}$ (B) $\frac{8}{3}$ (C) $\frac{3}{8}$ (D) $\frac{1}{2}$

38. The portion of the curve $y = x^2$ between $x = 0$ and $x = 2$, is rotated completely round the x –axis. Find the volume of the solid created in cubic units. (A) $\frac{30}{5}$ (B) $\frac{\pi}{5}$ (C) 32π (D) $\frac{32\pi}{5}$

39. The part of the curve $y = x^2$ from $x = 1$ to $x = 2$, is rotated completely round the y –axis. Find the volume of the solid generated (A) $\frac{93\pi}{5}$ (B) $\frac{9\pi}{5}$ (C) $\frac{5\pi}{9}$ (D) $\frac{92\pi}{5}$

40. Evaluate $\int e^{ax}dx$ (A) $e^{ax} + k$ (B) $\frac{e^{ax}}{a} + k$ (C) $ae^{ax} + k$ (D) $\frac{e^{ax}}{a}$

$9x^2+5$

$9x^2-10x$

SECTION B: THEORY

INSRUCTION: ANSWER ANY TWO QUESTIONS IN THIS SECTION

1 (a) Find the derivative of the function f if $f(x) = \frac{(3x-1)(x+5)}{2x^2}$

(b) A particle moves in a straight line so that its distance Smetres from a fixed point on the line at time t secs is giving by $S = t^4 + 3t^2$. Find the (i) velocity (ii) acceleration when $t = 1$

2. (a) Differentiate from First-Principle $y = 3x^3 - 5x^2 + 2$ at $x = 5$.

(b) Find the slope of the tangent to the curve $y = (x^2 - 2x + 1)(3x^3 - 5x^2 + 2)$ at $x = 2$.

3. (a) Evaluate $\int \sqrt{(x^3 - 5)}\, 3x^2 dx$

(b) Evaluate $\int cos5x\, dx$

Bio-Calculus

{Life of Calculus}

Solutions

① $f(x) = x^2 + 13x - 12$ and if $x = 3$ (Just put the value of $x = 3$)

$f(x) = 3^2 + 13(3) - 12$

$= 9 + 39 - 12 = \underline{\underline{36}}$

Ans = $\underline{\underline{B}}$

② $f(x) = (-x)^2 + 2x - 21$ and if $x = 2$ (Just put the value of $x = 2$)

$f(x) = (-2)^2 + 2(2) - 21$

$= 4 + 4 - 21 = \underline{\underline{-13}}$

Ans = not in the options

③ $f(x) = \cos x$ (The graph of $\cos x$ is symmetric on y-axis therefore $\cos x$ is an even function)

$\left.\begin{array}{l}\cos(x) = \frac{x}{r} \\ \cos(-x) = \frac{x}{r}\end{array}\right\}$ even function

is even : $f(-x) = f(x)$

$\cos x = -\cos x$

$f(x)$: $\cos x = -\cos x$

$f(-x)$: $-\cos x = \cos x$

$\therefore$ $\cos x$ is an even function

Ans = A

④ $f(x) = x^3 + 1$

$f(-x) = -x^3 + 1$

Ans = $\underline{\underline{D}}$

Neither odd nor even

⑤ $f(x) = \tan x$

$\left.\begin{array}{l}\tan x = y/x \\ \tan(-x) = \frac{-y}{x}\end{array}\right\}$ odd function

Ans = $\underline{\underline{B}}$

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$f(t) = 3t^3 + 2t$ at the point $t = 2$

$3(2)^3 + 2(2) = 28$ Ans = C

(7) $f(x) = \ln x = \frac{f'(x)}{f(x)} = \frac{\text{derivative of } x}{x} = 1/x$

Ans = B

(8) $\frac{d(e^{3x^2})}{dx}$ = find the derivative of the ~~power~~ exponential power and ~~used~~ use it to multiply the exponential

$3x^2 = 6x$ therefore $6x \times e^{3x^2} = 6x e^{3x^2}$

Ans = C.

(9) Evaluate $\lim_{x \to x_0} \frac{x^2 - x_0^2}{x - x_0} = \lim_{x \to x_0} \frac{(x - x_0)^2}{x - x_0} = \frac{(x - x_0)(x - x_0)}{x - x_0}$

$= \lim_{x \to x_0} \frac{x^2 - xx_0 - xx_0 + x_0^2}{x - x_0} = \frac{x^2 - 2xx_0 + x_0^2}{x - x_0}$

take the derivative $= \lim_{x \to x_0} \frac{2x - 2x + 2x_0}{x - x_0} = \lim_{x \to x_0} \frac{2x_0}{x - x_0}$

Apply your limit

$= \frac{2x_0}{x_0 - x_0} = 2x_0$ Ans = C

(10) The Question is not clear

(11) The integral $\int \tan x \, dx = \ln|\sec x| + c$

Ans = A

(12) $\int \sec x \tan x \, dx$ NB: Integration is the Reverse of Differentiation

In differentiation $\sec x' = \sec x \tan x$

therefore $\int \sec x \tan x \, dx = \sec x + c$ Ans = A

(13) $\int \sec^2 x \, dx = \tan x + c$ Ans = A

(14) $\int \sin x \, dx = -\cos x + c$ Ans = D

(15) when the gradient function of a curve, $\frac{dy}{dx}$, is given, we can find the equation of the curve. Thus, given $\frac{dy}{dx} = f(x) \Rightarrow y = \int f(x)$

therefore

$$6x + 2 = \int 6x + 2 \, dx$$

$$= \frac{6x^{1+1}}{1+1} + \frac{2x^{0+1}}{0+1} + c = \frac{6x^2}{2} + 2x + c = \underline{\underline{3x^2 + 2x + c}}$$

where c is an arbitrary constant which must be determined

The curve passes through the point $P(\overset{x}{1}, \overset{y}{3})$.

$x = 1, \quad y = 3$

$y = 3x^2 + 2x + c$

$3 = 3(1)^2 + 2(1) + c$

$3 = 3 + 2 + c$

$3 - 3 - 2 = c$

$c = -2$ substitute the value of c in your answer bros

Therefore, $\underline{\underline{y = 3x^2 + 2x - 2}}$ Ans = $\underline{\underline{C}}$

(16) find $\frac{dy}{dx}$ if $y = x^{2x}$

$y = x^{2x}$ take the natural log of both side i.e Same $\boxed{\ln = \log}$

$\ln y = \ln x^{2x}$

$\ln y = 2x \ln x$

$\boxed{\ln y' = \frac{1}{y}\frac{dy}{dx}}$

$2x \ln x$ (use product rule)

Therefore $\ln x^{2x}$ is same as

$\log_e x^{2x}$

Bros the 2x will move back

$2x \log_e x$ same as

$2x \ln x$

$2x \ln x$
$u \quad v$

$u = 2x, \quad \frac{du}{dx} = 2$

$v = \ln x \quad \frac{dv}{dx} = 1/x.$

$u\frac{dv}{dx} + v\frac{du}{dx}$

$2\cancel{x} \times \frac{1}{\cancel{x}} + \ln x \times 2$

$2 + 2\ln x$ $\boxed{2 \text{ is common}}$

$2(\ln x + 1)$ go back to your ~~question~~ solution

$\ln y = 2x \ln x$

$\frac{1}{y} \cdot \frac{dy}{dx} = 2(\ln x + 1)$ to make $\frac{dy}{dx}$ subject of the formular multiply through by y

$y \times \frac{1}{y}\frac{dy}{dx} = 2y(\ln x + 1)$

$\frac{dy}{dx} = 2y(\ln x + 1)$ Remember $y = x^{2x}$

$\frac{dy}{dx} = 2x^{2x}(\ln x + 1)$ Ans = C

(17) Ans = B

(18) Ans = D

(19) $\int f(x) f'(x)\,dx$ Integrate by substitution

let $u = f(x)$

$\frac{du}{dx} = f'(x), \quad dx = \frac{du}{f'(x)}$

Substitute the value in your question

$\int u f'(x) \frac{du}{f'(x)} = \int u\,du$ ~~into~~ Integrate now

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$\int u\,du$ using $\frac{x^{n+1}}{n+1}$ $\therefore$ $\frac{u^{n+1}}{n+1}$

$\frac{u^{1+1}}{1+1} = \frac{u^2}{2}$ $\qquad u = f(x)$

$\frac{u^2}{2} = \frac{(f(x))^2}{2} + K$ $\qquad$ Ans $= A$

(20) $\int \frac{\sin x}{1+\cos x}\,dx$ using substitution method

$u = 1 + \cos x$

$\frac{du}{dx} = -\sin x$ $\qquad$ make dx subject of formular

$dx = \frac{du}{-\sin x}$ $\therefore$

$\int \frac{\sin x}{1+\cos x}\,dx = \int \frac{\sin x}{u} \cdot \frac{du}{-\sin x} = -\int \frac{\sin x}{u} \cdot \frac{du}{\sin x}$

$= -\int \frac{1}{u}\,du$ $\qquad$ sir, remember $\frac{1}{u} = \ln u$

so, $-\ln u = -\ln(1+\cos x) + K$ $\qquad$ Ans $= A$

(21) NB! Integration is the reverse of Differentiation

$\sin^{-1}x = \frac{1}{\sqrt{1-x^2}}$ $\qquad$ therefore $\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1}x + c$

so. $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + c$

therefore $x = a\sin\theta$ $\qquad$ Ans $= C$

(22) using Exponential Rule which state that $\frac{d}{dx} a^x$ is $a^x \ln a$

where $a = 3$

so Bros, remember $\boxed{a^x = a^x \ln a}$

therefore, $3^x = \underline{3^x \ln 3}$ Ans = A

(23) $\lim_{x \to -1} x^2 - 5x + 2 = \lim_{x \to -1} x^2 - 5 \lim_{x \to -1} x + 2 = (-1)^2 - 5(-1) + 2$

$= 1 + 5 + 2 = 8$

Put the value of $x = -1$

Ans = B

(24) $\frac{dy}{dx}$ if $x = 1 + t^2$ and $y = 1/t$ at point $t = 2$

Using parametric formular $\boxed{\frac{dy}{dx} = \frac{dy}{dt} \Big/ \frac{dx}{dt}}$

$\frac{dx}{dt} = 2t$, $\frac{dy}{dt} = \frac{-1}{t^2}$

using the formular $\boxed{\text{i.e, } y = 1/t = t^{-1} = -t^{-1-1} = -t^{-2} = \frac{1}{-t^2} = \frac{-1}{t^2}}$

$\frac{\frac{-1}{t^2}}{\frac{2t}{1}} = \frac{-1}{t^2} \times \frac{1}{2t} = \frac{-1}{2t^3} = \frac{-1}{2(2)^3} = \frac{-1}{}$

(25) $y = \frac{\sqrt{3x+1}}{4x}$ $\frac{u}{v}$ using Quotient Rule

$u = \sqrt{3x+1}$, $\frac{du}{dx} = \sqrt{3x+1} = (3x+1)^{1/2} = \frac{3}{2}(3x+1)^{-1/2} = \frac{3}{2(3x+1)^{1/2}}$

$v = 4x$ $\frac{dv}{dx} = 4$

$$\boxed{\frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}} = \frac{4x \times \frac{3}{2\sqrt{3x+1}} - \sqrt{3x+1} \times 4}{(4x)^2} = \frac{\frac{2x \times 3}{\sqrt{3x+1}} - 4\sqrt{3x+1}}{16x^2}$$

multiply through by $\sqrt{3x+1}$

$$\frac{\left(\frac{6x}{\sqrt{3x+1}} \times \sqrt{3x+1}\right) - 4\sqrt{3x+1} \times \sqrt{3x+1}}{16x^2 \times \sqrt{3x+1}} = \frac{6x - 4(3x+1)}{16x^2\sqrt{3x+1}} = \frac{6x - 12x - 4}{16x^2\sqrt{3x+1}}$$

$$\frac{-6x-4}{16x^2\sqrt{3x+1}} = \frac{-2(3x+2)}{16x^2\sqrt{3x+1}} = -\frac{3x+2}{8x^2\sqrt{3x+1}} = -\frac{3x+2}{8x^2(3x+1)^{\frac{1}{2}}}$$

(26) $y = f(x)$ then $y + \Delta y = f(x+\Delta x)$

Ans = B//

(27) y'' of $y = 2x^3 - 3x^2 + 1$ $\boxed{y'' = \frac{d^2y}{dx^2}}$

so, $\frac{dy}{dx} = 6x^2 - 6x$

$\frac{d^2y}{dx^2} = 12x - 6$ Ans = B//

if $x^2 - xy = 1$, $\cancel{\frac{dy}{dx}}$ Differentiate by implicit

$2x - y - x\frac{dy}{dx} = 0$

$-x\frac{dy}{dx} = y - 2x$ divide ~~through~~ by $-x$

$\frac{dy}{dx} = \frac{y-2x}{-x}$ Ans = A//

(28) $f'(x)$ for $f(x) = e^{2x+5}$

using chain Rule $\left| \frac{dy}{du} \times \frac{du}{dx} \right|$

$y = e^{2x+5}$, $u = 2x+5$, $\frac{du}{dx} = 2$

$y = e^u$ $\frac{dy}{du} = e^u$

$= e^u \times 2 = 2e^u = 2e^{2x+5}$ Ans = A//

(29) Repeated Question No 25

(30) $y = \frac{f(x) - u}{g(x) - v}$ using Quotient Rule $\left[\frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2} \right\}$

$u = f(x)$, $\frac{du}{dx} = f'(x)$

$v = g(x)$ $\frac{dv}{dx} = g'(x)$ therefore $\frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$

Ans = A//

31) $\frac{d}{dx}(\ln(3x+2))$, that is $y = \ln(3x+2)$ using Chain Rule

$u = 3x+2$, $\frac{du}{dx} = 3$ $\boxed{\ln u = \frac{1}{u}}$

$y = \ln u$, $\frac{dy}{du} = \frac{1}{u}$

$\frac{1}{u} \times 3 = \frac{3}{u} = \frac{3}{3x+2}$ Ans = A

32) $\int \cosh x \, dx = \sinh x + K$ Ans = B

33) $x = t^2$, $y = t^3$ solve by Parametric $\left[\frac{dy}{dt} \Big/ \frac{dx}{dx}\right]$

$\frac{dx}{dt} = 2t$, $\frac{dy}{dx} = 3t^2$ therefore $\frac{3t^2}{2t} = \frac{3}{2}t$

Ans = A

34) D $\boxed{\text{A, B \& C are all true in Trignometry function}}$

35) $y = \sinh\frac{x}{2}$ find the derivative of $\boxed{\frac{x}{2} = \frac{1}{2}}$

~~using~~ chain Rule

$u = \frac{x}{2}$, $\frac{du}{dx} = \frac{1}{2}$ therefore, $\cosh u \times \frac{1}{2}$

$y = \sinh u$ $\frac{dy}{du} = \cosh u$ $\frac{1}{2}\cosh u = \frac{1}{2}\cosh\frac{x}{2}$

Ans = A

36) $S = t^2 - 5t + 6$ $\boxed{\frac{ds}{dt} = v}$

$\frac{ds}{dt} = 2t - 5$

at rest = 0

$2t - 5 = 0$.

$\frac{2t}{2} = \frac{5}{2}$ $t = 5/2$ sec Ans = A

37) B

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(37) $y = x^2$ between $x = 0$ and $x = 2$

our volume is $\boxed{V = \int_a^b \pi y^2 \, dx}$

so,

$$\pi \int_0^2 (x^2)^2 \, dx = \pi \int_0^2 x^4 \, dx \quad \text{Integrate}$$

$$= \pi \left[\frac{x^5}{5} \right]_0^2 = \pi \left[\frac{2^5}{5} \right] - \pi \left[\frac{0^5}{5} \right] = \frac{32\pi}{5}$$

Ans = D

(38) $y = x^3$ from $x = 1$ to $x = 2$

Since, they round the y-axis and are limits of y-corresponding to $x = 1$, $x = 2$. we must also express the integrand in terms of y.

our volume is $\boxed{V = \int_a^b \pi x^2 \, dy}$

$y = x^3$, let $x = 1$, $y = 1^3 = 1$ | $y = x^3$, let $x = 2$, $y = 2^3 = 8$

Remember $y = x^3$ (take the cube root of both side)

$x = \sqrt[3]{y}$, $x = y^{1/3}$

$$V = \int_1^8 \pi (y^{1/3})^2 dy = \pi \int_1^8 y^{2/3} dy = \pi \left[\frac{3y^{5/3}}{5} \right]_1^8 = \pi \left[\frac{3 \times 32}{5} \right] - \pi \left[\frac{3 \times 1}{5} \right]$$

$$= \frac{96\pi}{5} - \frac{3\pi}{5} = \frac{96\pi - 3\pi}{5} = \frac{93\pi}{5}$$

Ans = A

(39) $\int e^{ax} dx$

$\boxed{\text{NB! } \int e^x = e^x + k \text{ but } \int e^{ax} = \frac{e^{ax}}{a} + k}$

$$\int e^{ax} dx = \frac{e^{ax}}{a} + k$$

Ans = B

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SECTION B

(1)

(a) $f(x) = \frac{(3x-1)(x+5)}{2x^2}$ $\boxed{\text{Expand the numerator}}$

$= \frac{3x^2+15x-x-5}{2x^2} = \frac{3x^2+14x-5}{2x^2}$ $\boxed{\text{Share the Denominator for each Numerator}}$

$= \frac{3x^2}{2x^2} + \frac{14x}{2x^2} - \frac{5}{2x^2} = \frac{3}{2} + \frac{7}{x} - \frac{5}{2x^2} = \frac{3}{2} + 7x^{-1} - \frac{5}{2}x^{-2}$

$f'(x) = \frac{d}{dx}\left(\frac{3}{2} + 7x^{-1} - \frac{5}{2}x^{-2}\right) = 0 - 7x^{-2} + \frac{10}{2}x^{-3} = \frac{-7}{x^2} + \frac{5}{x^3}$

(b) $S = t^4 + 3t^2$. find the (i) velocity (ii) acceleration when $t=1$

$\boxed{\text{Velocity} = \frac{ds}{dt}}$ $\boxed{\text{acceleration} = \frac{dv}{dt}}$

— to find velocity

$V = \frac{ds}{dt} = 4t^3 + 6t$ when $t=1$

$V = 4t^3 + 6t$, $v = 4(1)^3 + 6(1)$ $V = 4+6 = 10m/s$

— to find acceleration

$a = \frac{dv}{dt} = 4t^3 + 6t$ when $t=1$

$\frac{dv}{dt} = 12(t)^2 + 6$, $a = 12(1)^2 + 6$, $a = 12+6 = 18ms^{-2}$

(2)

from first-principle $y = 3x^3 - 5x^2 + 2$ at $x=5$

Add Change to both side $\boxed{\Delta}$

$y + \Delta y = 3(x+\Delta x)^3 - 5(x+\Delta x)^2 + 2$

expand the bracket first

$(x+\Delta x)^3$ using Binomial theorem

$\boxed{x^n + {}^nC_r x^{-}(\Delta x)^r}$

n = Power
r = Positive integral starting from $1, 2, 3, \ldots$
$-$ = difference of n & r

$x^3 + {}^3C_1 x^2 \Delta x + {}^3C_2 x \Delta x^2 + {}^3C_3 \Delta x^3$

$\boxed{{}^3C_1 = 3}$ $\boxed{{}^3C_2 = 3}$ $\boxed{{}^3C_3 = 1}$

$\boxed{x^3 + 3x^2\Delta x + 3x\Delta x^2 + \Delta x^3}$

$(x+\Delta x)^2 = (x+\Delta x)(x+\Delta x) = x^2 + x\Delta x + x\Delta x + \Delta x^2$

$\boxed{x^2 + 2x\Delta x + \Delta x^2}$

therefore

$y + \Delta y = 3(x+\Delta x)^3 - 5(x+\Delta x)^2 + 2$

$y + \Delta y = 3(x^3 + 3x^2\Delta x + 3x\Delta x^2 + \Delta x^3) - 5(x^2 + 2x\Delta x + \Delta x^2) + 2$

$y + \Delta y = 3x^3 + 9x^2\Delta x + 9x\Delta x^2 + 3\Delta x^3 - 5x^2 - 10x\Delta x - 5\Delta x^2 + 2$

Subtract y from both side

$y + \Delta y - y = 3x^3 + 9x^2\Delta x + 9x\Delta x^2 + 3\Delta x^3 - 5x^2 - 10x\Delta x - 5\Delta x^2 + 2 - y$

Substitute the value of y

$\Delta y = 3x^3 + 9x^2\Delta x + 9x\Delta x^2 + 3\Delta x^3 - 5x^2 - 10x\Delta x - 5\Delta x^2 + 2 - (3x^3 - 5x^2 + 2)$

$\Delta y = 3x^3 + 9x^2\Delta x + 9x\Delta x^2 + 3\Delta x^3 - 5x^2 - 10x\Delta x - 5\Delta x^2 + 2 - 3x^3 + 5x^2 - 2$

$\Delta y = 3x^3 + 9x^2\Delta x + 9x\Delta x^2 + 3\Delta x^3 - 5x^2 - 10x\Delta x - 5\Delta x^2$

$\Delta y = 9x^2\Delta x + 9x\Delta x^2 + 3\Delta x^3 - 10x\Delta x - 5\Delta x^2$

Divide through by Δx

$$\frac{\Delta y}{\Delta x} = \frac{9x^2\Delta x}{\Delta x} + \frac{9x\Delta x^2}{\Delta x} + \frac{3\Delta x^3}{\Delta x} - \frac{10x\Delta x}{\Delta x} - \frac{5\Delta x^2}{\Delta x}$$

$$\frac{\Delta y}{\Delta x} = 9x^2 + 9x\Delta x + 3\Delta x^2 - 10x - 5\Delta x$$

take limit

$$\lim_{\Delta x \to 0} \frac{dy}{dx} = 9x^2 - 10x$$

at $x = 5$: $9x^2 - 10x = 9(5)^2 - 10(5) = 225 - 50$

$= 175$

b) $y = (x^2 - 2x + 1)(3x^3 - 5x^2 + 2)$ at $x = 2$

so the point we are targeting is $(2, 6)$

$y = ((2)^2 - 2(2) + 1)(3(2)^3 - 5(2)^2 + 2)$

$y = (4 - 4 + 1)(24 - 20 + 2)$

$y = (1)(6)$, $y = 6$

To find the slope of the tangent line, differentiate and then plug in $x = 2$.

$y = \underbrace{(x^2 - 2x + 1)}_{u}\underbrace{(3x^3 - 5x^2 + 2)}_{v}$

using Product Rule

$$\boxed{u\frac{du}{dx} + v\frac{du}{dx}}$$

$u = x^2 - 2x + 1$, $\frac{du}{dx} = 2x - 2$

$v = 3x^3 - 5x^2 + 2$ $\frac{dv}{dx} = 9x^2 - 10x$

$(x^2 - 2x + 1)(9x^2 - 10x) + (3x^3 - 5x^2 + 2)(2x - 2)$

$9x^4 - 18x^3 + 9x^2 - 10x^3 + 20x^2 - 10x + 6x^4 - 10x^3 + 4x - 6x^3 + 10x^2 - 4$

$15x^4 - 44x^3 + 39x^2 - 6x - 4$

so plug $x = 2$

$15(2)^4 - 44(2)^3 + 39(2)^2 - 6(2) - 4$

$240 - 352 + 156 - 12 - 4 = 28$

So, the slope of the tangent line is 28

- If you are ask to find the equation of ~~the~~ tangent line to a curve.

Use the Point-slope formula to compute ~~to compute~~ the equation

$y - y_1 = m(x - x_1)$ (x_1, y_1) are the original coordinates

therefore, $y - 6 = m(x - 2)$ $m = 28$

$y - 6 = 28(x - 2)$

$y - 6 = 28x - 56$

$y = \underline{28x - 50}$

(3)

Evaluate $\int \sqrt{(x^3-5)}\, 3x^2\, dx$

Choose the inside function as u

$u = x^3 - 5$, $\quad \frac{du}{dx} = 3x^2$, $\quad dx = \frac{du}{3x^2}$

$\int \sqrt{u}\, 3x^2 \frac{du}{3x^2} = \int \sqrt{u}\, du = \int u^{1/2}\, du = \frac{u^{1/2+1}}{1/2+1} + c$

$= \frac{u^{3/2}}{3/2} + c = \frac{2}{3} u^{3/2} + c = \frac{2}{3}(x^3-5)^{3/2} + c$

where $u = x^3 - 5$

(b)

(b) Evaluate $\int \cos 5x\, dx$ $\quad u = 5x$, $\frac{du}{dx} = 5$, $dx = \frac{du}{5}$

integration of $\int \cos u = \sin u$

$\int \cos u \frac{du}{5} = \frac{1}{5}\int \cos u\, du = \frac{1}{5}\sin u + c = \frac{1}{5}\sin 5x + c$

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